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Scattering theory for differential operators, I, operator theory

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§ 1. Introduction.

The present paper is intended to be the first of a series of papers aimed at dealing with a spectral and scattering theory for some partial differential operators by application of the so-called abstract stationary method. We take the attitude of studying problems in operator-theoretical terms as far as possible and then handling differential operators by applying the obtained results.

Some problems considered in the mathematical theory of scattering are: i) to investigate the structure of the absolutely continuous spectrum of a perturbed operator; ii) to prove the existence and the completeness of wave operators; iii) to establish the discreteness, as defined in § 5, of the singular spectrum; and iv) to construct eigenfunction expansions. Among many works concerning these problems we only mention a work of Ikebe in 1960 ([**5**]) and a group of more recent works on the abstract stationary method[1] ([**11**], [**9**], [**14**]). In [**5**] Ikebe treated the Schrödinger operator $-\Delta+q(x)$ by the integral equation method under the main assumption that $q(x)=O(|x|^{-\delta})$, $\delta>2$, as $|x|\to\infty$. With the aid of a theorem of Kato [**7**] concerning the growth property of the solution of $-\Delta u+qu=\lambda u$, he solved i)-iv) with a sharper result that there is no singular continuous spectrum except for non-positive eigenvalues. (The method was later applied to exterior problems in [**17**], [**6**], etc.) On the other hand, it was shown in [**11**] etc. that problems i) and ii) (and iv) partly) can be handled by the abstract stationary method. In particular, it was shown by Kato [**9**] that problems i) and ii) for Schrödinger operators can be solved for $\delta>1$ and that the sharper result as Ikebe's holds for $\delta>5/4$. Some more results were announced in [**14**].

Recently, S. Agmon investigated the spectral problem of differential operators by a new method based on a weighted elliptic estimate and an-

1) For an overall exposition of the scattering theory with an extensive list of literatures, the reader is referred to Kato [**10**].