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Invariants of finite abelian groups

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Introduction.

Let k be a field and let G be a finite group. Let V be a (finite dimensional) kG-module, i.e., a representation module of G over k. Then G acts naturally on the quotient field F of the symmetric algebra $S(V)$ of V as k-automorphisms. We denote the field F with this action of G by $k(V)$.

An extension L/k is said to be rational if L is finitely generated and purely transcendental over k.

To simplify our notation, we say that a triple $\langle k, G, V\rangle$ has the property (R) if $k(V)^G/k$ is rational. Especially, if V is the regular representation module of G, i.e., if $V=kG$, then we use $\langle k, G\rangle$ instead of $\langle k, G, V\rangle$.

The following problem is the classical and basic one (e.g. [**11**]).

Does $\langle k, G, V\rangle$ *have the property* (R)?

It is well known that the answer to the problem is affirmative in each of the following cases:

(i) G is the symmetric group, k is any field and $V=kG$.

(ii) G is an abelian group of exponent e and k is a field whose characteristic does not divide e and which contains a primitive e-th root of unity. (Fisher [**5**], etc.)

(iii) G is a p-group and k is a field of characteristic p. (Kuniyoshi [**6**], etc.)

(iv) k is a field of characteristic 0 and G is a finite group generated by reflections of a k-module V (Chevalley [**2**]).

However the problem has been kept open even in the case where G is abelian and k is an algebraic number field.

K. Masuda proved in [7] and [8] that $\langle Q, G\rangle$ has the property (R) when G is a cyclic group of order $n\leqq 7$ or $n=11$, and reduced the problem to the one on integral representations, in case G is a cyclic group of order p. Recently R. G. Swan [**15**] showed, using the Masuda's result, that $\langle Q, G\rangle$ does not have the property (R) when G is a cyclic group of order $p=47, 113, 233, \cdots$.

In this paper we will refine the Masuda-Swan's method and will give some further consequences on the problem in case G is abelian.