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Theta series and automorphic forms on GL_2

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The main purpose of the present paper is to give another proof of Jacquet-Langlands [5, Th. 14.4], the assertion of which is the following.

Let $\mathcal{K}$ be a division quaternion algebra over a global field F. To every irreducible admissible representation π of the Hecke algebra $\mathcal{H}(\mathcal{K}_A^\times)$, we can make correspond an irreducible admissible representation π^* of the Hecke algebra $\mathcal{H}(GL_2(\boldsymbol{A}))$ so that, if π is a constituent of the representation of $\mathcal{H}(\mathcal{K}_A^\times)$ in $\mathcal{A}(\eta, \mathcal{K}_A^\times)$ (the space of automorphic forms on $\mathcal{K}_A^\times$ with a character η), then π^* is a constituent of the representation of $\mathcal{H}(GL_2(\boldsymbol{A}))$ in $\mathcal{A}_0(\eta, GL_2(\boldsymbol{A}))$ (the space of cusp forms on $GL_2(\boldsymbol{A})$ with a character η) under the condition that the component π_v of π is infinite dimensional for all places v of F unramified in $\mathcal{K}$.

In view of various ideas in Jacquet-Langlands [5], and also of Shalika-Tanaka [7] and Weil [11], we find it natural to consider theta series made of Weil representation of $SL_2(\boldsymbol{A})$ in the Schwartz space on $\mathcal{K}_{\boldsymbol{A}}$, and in order to construct an irreducible subspace of $\mathcal{A}_0(\eta, GL_2(\boldsymbol{A}))$ from the space of theta series, to make use of a spherical function associated with automorphic forms on $\mathcal{K}_A^\times$. In this way we obtain a proof of the above theorem, somewhat more direct than the original one, under a weaker condition that π is not one-dimensional (in substance our proof is quite similar to that of [5, Th. 13.1]). The main theorem in our formulation is stated as Theorem 1 (§5, No. 12). Applied to the holomorphic automorphic forms, it gives a generalization of Eichler [1, 2]. It is stated as Theorem 2 (§6, No. 5).

For convenience sake we summarize in §1-§3 generalities on admissible representations, theta series, automorphic forms and spherical functions.

§1. Admissible representation of GL_2.

1. Definition (non-archimedean case). In No. 1—No. 4, F will be a non-archimedean local field. By an *admissible representation* π of $GL_2(F)$ we understand a representation π of $GL_2(F)$ in a vector space $\mathcal{V}$ over $\boldsymbol{C}$ satisfying the following conditions.

(1.1) For any $x \in \mathcal{V}$, the group of elements g in $GL_2(F)$ such that