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A relative Hodge-Kodaira decomposition

By Daisuke FUJIWARA

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§1. Introduction.

Let X be an $m+n$ dimensional oriented compact C^{∞} Riemannian manifold and $\Omega^{p}(X)$ be the space of smooth p-forms on X. Celebrated Hodge theorem says that every cohomology class $H^{p}(X)$ of de Rham is canonically represented by a harmonic p-form (cf. [2] and [5]). The aim of this note is to prove an analogy of this theorem of Hodge also for the cohomology group $H^{p}(X, Y)$ relative to m-dimensional submanifold $Y\subset X$. More precisely, let Y be an m-dimensional compact oriented submanifold of X and $\Omega^{p}(Y)$ be the space of smooth p-forms of Y. The relative cohomology group $H^{*}(X, Y)$ is the cohomology group of the complex $\Omega^{*}(X, Y)$ defined by the exact sequence of complexes

$$0\longrightarrow\Omega^{*}(X, Y)\longrightarrow\Omega^{*}(X)\overset{\iota}{\longrightarrow}\Omega^{*}(Y)\longrightarrow 0 \tag{1.1}$$

where ι is the restriction mapping. Kodaira [5] proved that every cohomology class of $H^{p}(X, Y)$ can be represented by a square summable harmonic p-forms on open submanifold $X-Y$ of X (cf. [2]). However, this is not convenient when one wants to deal with the long exact sequence

$$\begin{aligned} (1.2)\qquad & 0\longrightarrow H^{0}(X, Y)\longrightarrow H^{0}(X)\longrightarrow H^{0}(Y)\longrightarrow \\ & \longrightarrow H^{1}(X, Y)\longrightarrow H^{1}(X)\longrightarrow H^{1}(Y)\longrightarrow \\ & \qquad\cdots\cdots\cdots\cdots\cdots\cdots \\ & \longrightarrow H^{p}(X, Y)\longrightarrow H^{p}(X)\longrightarrow H^{p}(Y)\longrightarrow \\ & \qquad\cdots\cdots\cdots\cdots\cdots\cdots . \end{aligned}$$

In this note we prove the following facts:

(i) Every cohomology class of $H^{p}(X, Y)$ can be represented by a current α on X which satisfies the equation

$$\Delta(1+\Delta)^{a}\alpha=0 \qquad \text{on } X-Y \tag{1.3}$$

and $\alpha|_{Y}=0$, where a is the integral part of $n/2$.

(ii) Every cohomology class of $H^{p}(X, Y)$ can be represented by a current β on X which is harmonic in $X-Y$ and has singularity on Y.