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Approximation of nonlinear semigroups and evolution equations

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§ 1. Introduction.

Consider a sequence of abstract Cauchy problems

$$(d/dt)u_n(t) \in A_n(t)u_n(t) \qquad (t \geqq 0),\ u_n(0) = x_n,\ n = 0, 1, 2, \cdots \tag{1.1}$$

in an arbitrary Banach space X. Here $A_n(t)$ is for each t a multi-valued function defined on a subset of X. We shall show that under suitable hypotheses, if x_n converges to x_0 and if $A_n(t)$ converges to $A_0(t)$ (in a sense to be made precise below), then $u_n(t)$ converges to $u_0(t)$.

We first deal with the case when the multi-valued function A_n does not depend on t and determines a strongly continuous semigroup of Lipschitzian operators on a subset of X. In Section 3, using a generation theorem of Crandall and Liggett [4], we obtain nonlinear generalizations of the Trotter-Neveu-Kato approximation theorem for semigroups. These extend results of a number of authors, including Brezis and Pazy [2], Mermin [14], Miyadera [15], and Miyadera and Ôharu [17]. Moreover, our result is best possible in the sense that our sufficient condition is necessary in the linear case.

In Section 4 we establish existence and uniqueness criteria for a special class of time dependent multi-valued Cauchy problems of the form

$$(d/dt)u(t) \in A(t)u(t) \qquad (t \geqq 0),\quad u(0) = x \tag{1.2}$$

in a Hilbert space. We also prove an approximation theorem in this situation.

Finally, using existence theorems of Crandall and Liggett [4] and Martin [12], we establish in Section 5 approximation theorems for a class of problems of the form (1.1) in an arbitrary Banach space setting.

§ 2. Notation.

Let X be a Banach space with norm $\|\cdot\|$. When X is a Hilbert space, its inner product will be denoted by $\langle\cdot,\cdot\rangle$. "lim" [resp. "$w$-lim"] refers to limit in the norm [resp. weak] topology of X. $\mathcal{P}(X)$ denotes the set of all subsets of X. $\boldsymbol{R}$ denotes the real numbers, $\boldsymbol{R}^+$ the nonnegative reals, $\boldsymbol{Z}^+$ the non-