

Regular congruences on Croisot-Teissier and Baer-Levi semigroups

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Clifford and Preston have defined a class of Croisot-Teissier semigroups of type (p, q) which are simple with a minimal right ideal when $p = q$ ([1] 8.2). In this paper, we define a modified class of Croisot-Teissier semigroups (1.3) which are simple with a minimal right ideal for all $q \leq p$ (1.9). The generalized Baer-Levi semigroups ([1] 8.1) are shown to be right simple generalized Croisot-Teissier semigroups under the new definition (1.5).

In the concluding sections, we investigate group and band congruences on these semigroups. We find a set, E , which is contained in the kernel of every group congruence (2.3), and also find necessary and sufficient conditions for E to be the kernel of such a congruence (2.11). Using this result, we show that a Baer-Levi semigroup has a non-trivial group congruence if and only if $p > q$ (2.14).

Finally, we relate band congruences on simple semigroups with a minimal right ideal to the ordering of the \mathcal{L} -classes under the usual ordering (3.3), and after investigating this structure in Baer-Levi semigroups (3.5), we show they have only trivial band congruences. This is sufficient to show that the only regular congruences on Baer-Levi semigroups are group congruences (3.6).

The terminology and notation will be that of Clifford and Preston [1].

§1. Generalized Croisot-Teissier semigroups.

In this section we discuss a class of simple semigroups with a minimal right ideal, which are generalized Baer-Levi semigroups ([1] 8.1). Clifford and Preston ([1] 8.2) have defined Croisot-Teissier semigroups of type (p, q) which are simple with a minimal right ideal in the case $p = q$. We will modify their definition to obtain a class of generalized Croisot-Teissier semigroups of type (p, q) ($p \geq q$), each member of which is simple with a minimal right ideal.

(1.1) DEFINITION ([1] vol. II, p. 86). Let p and q be infinite cardinals