On the $K$-theoretic characteristic numbers of weakly almost complex manifolds with involution

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§ 0. Introduction.

In [7], tom Dieck has defined the equivariant unitary cobordism ring $U_G$ for any compact Lie group $G$. $U_G$-theory seems to be a strong tool in the theory of the differentiable transformation group.

We are concerned only with the case of $G = Z_2$, the cyclic group of order 2, and throughout in this paper, the letter $G$ stands for $Z_2$. Let $O^U(G)$ be the bordism ring of $U$-manifolds with involution. T. tom Dieck has shown that elements of $O^U(G)$ are detected by $G$-equivariant characteristic numbers. More precisely we construct a ring homomorphism

$\Phi : U_G^* \rightarrow \text{Inv. Lim. } R(G)[[t_1, \cdots, t_s]]$

and its localization

$\Phi_L : U_G^* \rightarrow \text{Inv. Lim. } Q[[t_1, \cdots, t_s]].$

Then the restriction of $\Phi$ on $U_G^n$ is injective. We shall recapitulate this fact in (1.1) for the sake of completeness, and we give the explicit form of $\Phi_L$ in (3.1) and its relation to $\Phi$ in (3.2).

As corollaries of (1.1) and (3.2), the following results will be proved in § 4.

THEOREM (0.1). Let $[M, T] \in O^U(G)$. The normal bundle $\nu_F$ of a connected component of the fixed point set $F$ in $M$ naturally has a complex structure. Assume the following two conditions:

(i) For each connected component $F$, $\nu_F$ is trivial,

(ii) $\dim \nu_F$ is independent of $F$ and equals a constant $n$.

Then $\sum [F] \in 2^n U$ and there are two elements of $U_*, [N]$ and $[L]$ such that

$[M, T] = [CP(1), \tau]^*[N] + [G, \sigma][L]$ in $O^U(G)$

where $[CP(1), \tau] \in O^U(G)$ is the class of $CP(1)$ with the involution $[z_1, z_2] \rightarrow [z_1, -z_2]$ and $[G, \sigma] \in O^U(G)$ is the class of $G$ with the natural involution $1 \rightarrow -1$.

THEOREM (0.2). Let $[M, T] \in O^U(G)$. If $M$ is a Kähler manifold, and $T$