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On the K-theoretic characteristic numbers of weakly almost complex manifolds with involution

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§ 0. Introduction.

In [7], tom Dieck has defined the equivariant unitary cobordism ring U_G for any compact Lie group G. U_G-theory seems to be a strong tool in the theory of the differentiable transformation group.

We are concerned only with the case of $G=Z_2$, the cyclic group of order 2, and throughout in this paper, the letter G stands for Z_2. Let $\mathcal{O}_*^U(G)$ be the bordism ring of U-manifolds with involution. T. tom Dieck has shown that elements of $\mathcal{O}_*^U(G)$ are detected by G-equivariant characteristic numbers. More precisely we construct a ring homomorphism

$$\Phi:\ U_G^* \longrightarrow \text{Inv. Lim. } R(G)[[t_1, \cdots, t_s]]$$

and its localization

$$\Phi_L:\ U_G^* \longrightarrow \text{Inv. Lim. } Q[[t_1, \cdots, t_s]]\,.$$

Then the restriction of Φ on U_G^n is injective. We shall recapitulate this fact in (1.1) for the sake of completeness, and we give the explicit form of Φ_L in (3.1) and its relation to Φ in (3.2).

As corollaries of (1.1) and (3.2), the following results will be proved in § 4.

THEOREM (0.1). *Let $[M, T]\in\mathcal{O}_*^U(G)$. The normal bundle ν_F of a connected component of the fixed point set F in M naturally has a complex structure. Assume the following two conditions:*

(i) *For each connected component F, ν_F is trivial,*

(ii) *$\dim_C\nu_F$ is independent of F and equals a constant n.*

Then $\sum[F]\in 2^nU$ and there are two elements of U_*, $[N]$ and $[L]$ such that

$$[M, T]=[CP(1), \tau]^n[N]+[G, \sigma][L] \qquad \text{in } \mathcal{O}_*^U(G)$$

where $[CP(1), \tau]\in\mathcal{O}_*^U(G)$ is the class of $CP(1)$ with the involution $[z_1, z_2]\to[z_1, -z_2]$ and $[G, \sigma]\in\mathcal{O}_*^U(G)$ is the class of G with the natural involution $1\to -1$.

THEOREM (0.2). *Let $[M, T]\in\mathcal{O}_*^U(G)$. If M is a Kähler manifold, and T*