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n-dimensional complex space forms immersed in $\left\{n+\frac{n(n+1)}{2}\right\}$-dimensional complex space forms

Dedicated to Professor Shigeo Sasaki on his 60th birthday

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§ 1. Introduction.

A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form*. A *Kaehler immersion* is an isometric immersion which is complex analytic. B. O'Neill ([2]) proved the following result.

Let M and $\tilde{M}$ be complex space forms of dimension n and $n+p$, respectively. If $p<\frac{n(n+1)}{2}$ and if M is a Kaehler submanifold of $\tilde{M}$, then M is totally geodesic in $\tilde{M}$.

He also gave the following example: There is a Kaehler imbedding of an n-dimensional complex projective space of constant holomorphic sectional curvature 1/2 into an $\left\{n+\frac{n(n+1)}{2}\right\}$-dimensional complex projective space of constant holomorphic sectional curvature 1. This shows that the dimensional restriction in the above result is the best possible.

We have proved in [1] the following result.

Let M be an n-dimensional complex space form of constant holomorphic sectional curvature c and $\tilde{M}$ be an $(n+p)$-dimensional complex space form of constant holomorphic sectional curvature $\tilde{c}$. If $p\geqq\frac{n(n+1)}{2}$ and if M is a Kaehler submanifold of $\tilde{M}$ with parallel second fundamental form, then either $c=\tilde{c}$ (i.e., M is totally geodesic in $\tilde{M}$) or $c=\tilde{c}/2$, the latter case arising only when $\tilde{c}>0$.

The purpose of this paper is to prove the following

THEOREM. *Let M be an n-dimensional complex space form of constant holomorphic sectional curvature c and $\tilde{M}$ be an $\left\{n+\frac{n(n+1)}{2}\right\}$-dimensional*

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