

## On the global existence of real analytic solutions of linear differential equations (I)\*

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### § 0. Introduction.

The purpose of this paper is to present global existence theorems for linear differential equations with constant coefficients which satisfy suitable regularity conditions. (Cf. conditions (1.1), (1.2) and Definition 4.2.) Our proof relies on the existence of good elementary solutions, the meaning of which is clarified in § 1 and § 4. Sato's theory of sheaf  $\mathcal{C}$  (Sato [2]~[5]) also plays an essential role in the course of the proof.

We remark that the problem of the global existence of real analytic solutions has remained unsolved because the topological structure of the space of real analytic functions on an open set  $\Omega$  in  $\mathbf{R}^n$  is a complicated one. (Cf. Ehrenpreis [1], Martineau [1].) In fact there has been no general result even when  $\Omega$  is convex; the only results hitherto known seem to be Theorems  $\alpha$  and  $\beta$ , which we list up below for the reader's convenience. We also note that during the preparation of this paper Professors E. De Giorgi and L. Cattabriga have informed the author that they have obtained the affirmative answer by the method of a priori estimate when  $\Omega = \mathbf{R}^2$ . (Cf. E. De

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