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Three-dimensional compact Kähler manifolds with positive holomorphic bisectional curvature

By Shoshichi KOBAYASHI*) and Takushiro OCHIAI**)

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§ 1. Introduction.

One of the most challenging problems in Riemannian geometry is to determine all compact Riemannian manifolds[1] with positive sectional curvature. As a special case, the following problem has been considered by Frankel [11].

Let M be a compact Kähler manifold of dimension n with positive sectional (or more generally, holomorphic bisectional) curvature. Is M necessarily biholomorphic to the complex projective space $P_n(\boldsymbol{C})$?

This is trivially true for $n=1$ since $P_1(\boldsymbol{C})$ is the only compact Riemann surface with positive first Chern class. The question has been answered affirmatively for $n=2$ by Frankel and Andreotti [11]; their proof depends on the classification of the rational surfaces. Recently, Howard and Smyth [18] have determined the compact Kähler surfaces of non-negative holomorphic bisectional curvature. In higher dimensions, this question has been answered affirmatively only under additional assumptions: 1) Pinching conditions (Howard [17]), or 2) Einstein-Kähler (Berger [2]) or constant scalar curvature (Bishop and Goldberg [4]).

The purpose of this paper is to answer the question above affirmatively for $n=3$, see Theorem 7.1. The proof given here leaves much to be desired, for it makes use of a difficult theorem of Aubin (see Lemma 7.3) and does not answer the following algebraic geometric question:

Let M be a compact complex manifold of dimension n with positive tangent bundle. Is M necessarily biholomorphic to $P_n(\boldsymbol{C})$?

This question, which is more general than the first one, has been answered affirmatively by Hartshorne [14] for $n=2$ by a purely algebraic method. It has been affirmatively answered also for the compact homogeneous complex manifolds [22] as well as for the complete intersection submanifolds of complex projective spaces [21]. In [21] we have shown that a 3-dimensional compact complex manifold M with positive tangent bundle admits a group

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1) All manifolds in this paper are connected unless otherwise stated.