J. Math. Soc. Japan
Vol. 24, No. 3, 1972

(3/2)-dimensional measure of singular sets of some Kleinian groups

By Tohru AKAZA

(Received Aug. 5, 1971)

§ 0. Introduction.

The writer showed the existence of Kleinian groups with fundamental domains bounded by four circles whose singular sets have positive 1-dimensional measure ([4]). Further in former paper [5], he investigated the properties about the computing functions on some Kleinian groups and gave the results with respect to the local properties of the singular sets of these groups by using the computing function. Now in the natural way the following problem arises; to what extent does the Hausdorff dimension of the singular sets of Kleinian groups climb up, when the number N of the boundary circles increases? It is conjectured and seems still open that the 2-dimensional measure of the singular sets of general finitely generated Kleinian groups is always zero ([1]).

The purpose of this paper is to show the existence of the Kleinian groups whose singular sets have positive (3/2)-dimensional measure. We shall state preliminaries and notations about Kleinian groups in § 1. In § 2 we shall introduce the subcomputing function on some Kleinian groups which have the analogous properties to the computing functions and investigate the relations between this function and the Hausdorff dimension of the singular set of some Kleinian group. By using this function we shall give the example of the Kleinian group whose singular set has positive (3/2)-dimensional measure in § 3.

§ 1. Preliminaries and notations.

1. Let B_0 be a domain bounded by $N\ (\geqq 3)$ mutually disjoint circles $\{H_i, H_i'\}_{i=1}^{p}$ and $\{K_j\}_{j=1}^{q}$ $(2p+q=N)$. Let S_i $(1\leqq i\leqq p)$ be a hyperbolic or loxodromic transformation which transforms the outside of H_i onto the inside of H_i'. Let S_j^* $(1\leqq j\leqq q)$ be an elliptic transformation of period 2 which transforms the outside of K_j onto the inside of K_j. Then $\{S_i\}_{i=1}^{p}\cup\{S_j^*\}_{j=1}^{q}$ generates a discontinuous group G with B_0 as a fundamental domain. We use the notation $\mathfrak{G}$ to denote the set of $\{S_i\}_{i=1}^{p}$, their inverses and $\{S_j^*\}_{j=1}^{q}$.