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A characterization of $PSL(2, 11)$ and S_5

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§ 1. Introduction.

The symmetric group S_5 of degree five and the two dimensional projective special linear group $PSL(2, 11)$ over the field of eleven elements are doubly transitive permutation groups of degree five and eleven, respectively, in which the stabilizer of two points is isomorphic to the symmetric group S_3 of degree three.

Let Ω be the set of points $1, 2, \cdots, n$, where n is odd. Let $\mathfrak{G}$ be a doubly transitive permutation group in which the stabilizer $\mathfrak{G}_{1,2}$ of the points 1 and 2 has even order and a Sylow 2-subgroup $\mathfrak{K}$ of $\mathfrak{G}_{1,2}$ is cyclic. In the case $\mathfrak{G}_{1,2}$ is cyclic, Kantor-O'Nan-Seitz and the author proved independently that $\mathfrak{G}$ contains a regular normal subgroup ([**5**] and [8]). In this paper we shall study the case $\mathfrak{G}_{1,2}$ is not cyclic. Let τ be the unique involution in $\mathfrak{K}$. By a theorem of Witt ([**10**, Th. 9.4]) the centralizer $C_{\mathfrak{G}}(\tau)$ of τ in $\mathfrak{G}$ acts doubly transitively on the set $\mathfrak{I}(\tau)$ consisting of points in Ω fixed by τ.

The purpose of this paper is to prove the following theorem.

THEOREM. *Let $\mathfrak{G}$, $\mathfrak{G}_{1,2}$, τ and $\mathfrak{I}(\tau)$ be as above. Assume that all Sylow subgroups of $\mathfrak{G}_{1,2}$ are cyclic, the image of the doubly transitive permutation representation of $C_{\mathfrak{G}}(\tau)$ on $\mathfrak{I}(\tau)$ contains a regular normal subgroup and that $\mathfrak{G}$ does not contain a regular normal subgroup. If $\mathfrak{G}$ has two classes of involutions, then $\mathfrak{G}$ is isomorphic to S_5 and $n=5$. If $\mathfrak{G}$ has one class of involutions and τ is not contained in the center of $\mathfrak{G}_{1,2}$, then $\mathfrak{G}$ is isomorphic to $PSL(2, 11)$ and $n=11$.*

In [**7**] we proved this theorem in the case that the order $\mathfrak{G}_{1,2}$ equals $2p$ for an odd prime number p.

Let $\mathfrak{X}$ be a subset of a permutation group. Let $\mathfrak{I}(\mathfrak{X})$ denote the set of all the fixed points of $\mathfrak{X}$ and let $\alpha(\mathfrak{X})$ be the number of points in $\mathfrak{I}(\mathfrak{X})$. The other notion is standard.

§ 2. On the degree of $\mathfrak{G}$.

Let $\mathfrak{G}$ be a doubly transitive permutation group on $\Omega=\{1, 2, \cdots, n\}$. Let $\mathfrak{G}_1$ and $\mathfrak{G}_{1,2}$ be the stabilizers of the point 1 and the points 1 and 2.