J. Math. Soc. Japan
Vol. 24, No. 3, 1972

The orbit-preserving transformation groups associated with a measurable flow

By Masasi KOWADA

(Received April 10, 1970)
(Revised Feb. 16, 1972)

§ 0. Introduction.

The purpose of this paper is to expose a general approach to the study of a measurable flow on a standard space with a probability measure.

For a given measurable flow $\mathfrak{T}$ we introduce the group $\mathfrak{G}$ of bimeasurable transformations which transform the orbits of the flow $\mathfrak{T}$ onto another orbits; we call such a transformation in $\mathfrak{G}$ an orbit-preserving transformation. Such group is related with many problems in the theory of flow. An orbit-preserving transformation yields a new flow, say a time changed flow of $\mathfrak{T}$ which is metrically isomorphic to the flow $\mathfrak{T}$. In this sense, such group $\mathfrak{G}$ makes the flow $\mathfrak{T}$ invariant and gives us informations about the geometry of trajectories of $\mathfrak{T}$. Moreover the group $\mathfrak{G}$ determines the subfamily of time change functions by which the time changed flows of $\mathfrak{T}$ are metrically isomorphic to $\mathfrak{T}$.

The notion of time change of flow was introduced by E. Hopf [**2**], and is extensively studied by G. Maruyama [**5**] and H. Totoki [**9**]. Our approach is different from them in the point of view of global analysis; for example, we ask in what class of flows the given flow $\mathfrak{T}$ is typical one.

Our method is worked by appealing to a cohomologous class of (one-) cocycles of the group $\mathfrak{G}$; the notion of additive functionals of a flow which was introduced by G. Maruyama is just cocycle of the group $\mathfrak{G}$ with respect to the additive group $\boldsymbol{R}$ of real numbers, and a time change function of flow is an inverse function in time variable of an additive functional, although our definition of time change functions is slightly different from his. These notions are defined in the sections 2 and 3.

The group $\mathfrak{G}$ contains important subgroups. As one of them we are concerned with the subgroup $\mathfrak{G}_s$ in the sections 3.3 and 3.4. The group $\mathfrak{G}_s$ consists of all transformations which transform each orbit of $\mathfrak{T}$ onto itself. This group $\mathfrak{G}_s$ is related with, for example, the time change of an analytic flow defined by a differential equation on the 2-dimensional torus which was