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$(p, q; r)$-absolutely summing operators

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By Mitjagin and Pełczyński [7] a linear operator T from a Banach space X into another Banach space Y is said to be (p, r)-absolutely summing, $1 \leqq p, r \leqq \infty$, if there is a constant ρ such that for every finite sequence $\{x_i\}_{1\leqq i\leqq n}$ of points in X the inequality

$$(1) \qquad (\sum_i \|Tx_i\|^p)^{1/p} \leqq \rho \sup_{\|a\|\leqq 1} (\sum_i |\langle x_i, a\rangle|^r)^{1/r}$$

holds. Here as usual, if $p=\infty$ (resp. $r=\infty$), the left (resp. right) hand side of (1) is replaced by $\sup_i \|Tx_i\|$ (resp. $\rho \sup_{\|a\|\leqq 1} (\sup_i |\langle x_i, a\rangle|)$). The notation $\sup_{\|a\|\leqq 1}$ means the supremum taken over all the elements a of the weakly compact unit ball of the dual space X* of X. The theory of (p, r)-absolutely summing operators is a unified theory of various important classes of operators in connection with the classes of nuclear and Hilbert-Schmidt operators.

In this paper we shall define $(p, q; r)$-absolutely summing operator, generalizing (p, r)-absolutely summing operator, inspired from the theory of Lorentz space $l_{p,q}$ of sequences. The aim of this paper is to develop the theory of this operator. In § 1 we discuss the basic properties of the class of $(p, q; r)$-absolutely summing operators as a Banach ideal. In § 2 we deal with the composition of these absolutely summing operators. We give there a generalization of the classical theorem that the composition of two Hilbert-Schmidt operators is nuclear. When the Banach spaces considered as domain and range are particular, for instance Hilbert spaces, some of Banach ideals of absolutely summing operators may happen to coincide. We shall state these facts in § 3 and § 4. We also investigate there the mean spaces (Lions-Peetre [6]) of Banach ideals of absolutely summing operators.

§ 1. $(p, q; r)$-absolutely summing operator.

Let X and Y be Banach spaces and let B(X, Y) be all the bounded linear operators from X into Y.

DEFINITION 1. Let $1 \leqq p, q, r \leqq \infty$. Let $\{x_i\}_{1\leqq i\leqq n}$ be any finite sequence of points in X, and $\{\|Tx_i\|_*\}$ be the non-increasing rearrangement of $\{\|Tx_i\|\}$.