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On transcendency of special values of arithmetic automorphic functions

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§ 1. Introduction.

Let Γ be the modular group $SL(2, \boldsymbol{Z})$ and $\tilde{\Gamma}=GL^{+}(2, \boldsymbol{Q})$. Let H be the complex upper half plane $\{z \in \boldsymbol{C};\ \mathrm{Im}\, z>0\}$. We define the action of an element $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of $GL^{+}(2, \boldsymbol{R})$ on H by

$$z \longmapsto \frac{az+b}{cz+d}$$

for $z \in H$. Then Γ and $\tilde{\Gamma}$ operate on H. Let $J(z)$ be the standard modular function of level one. Then the classical theory of complex multiplication shows:

THEOREM C. *If $z \in H$ is fixed by some non-scalar element of $\tilde{\Gamma}$, z is an algebraic number and $J(z)$ generates an abelian extension of $\boldsymbol{Q}(z)$.*

On the other hand, T. Schneider obtained the following theorem:

THEOREM T. *Let $z \in H$ be an algebraic number. Suppose that z is not fixed by any non-scalar element of Γ. Then $J(z)$ is a transcendental number.*

In this paper, we shall give a generalization of Theorem T.

Let B be an indefinite quaternion algebra over the rational number field $\boldsymbol{Q}$, $\mathcal{O}$ a maximal order of B, Γ the group of all the units of $\mathcal{O}$ of reduced norm one, and $\tilde{\Gamma}$ the group of all the invertible elements of B with positive reduced norm. Now we fix an irreducible representation χ of B into $M_2(\boldsymbol{R})$ so that the image $\chi(B)$ is contained in $M_2(\bar{\boldsymbol{Q}})$, where $\bar{\boldsymbol{Q}}$ is the algebraic closure of $\boldsymbol{Q}$ in $\boldsymbol{C}$. Then we may regard Γ and $\tilde{\Gamma}$ as subgroups of $GL^{+}(2, \boldsymbol{R})$ acting on H. As a generalization of the function J, G. Shimura has constructed a holomorphic map φ from H into a projective space $\boldsymbol{P}^{l}$, satisfying the following conditions (cf. Shimura [4], § 9): (i) φ induces a biregular isomorphism from $\Gamma\backslash H$ onto an algebraic curve in $\boldsymbol{P}^{l}$; (ii) if z is fixed by some non-scalar element of $\tilde{\Gamma}$, $\varphi(z)$ generates an abelian extension over a certain imaginary quadratic field. We shall call the map φ the *Shimura map*.

Now our main result can be stated as follows:

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