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Periodic maps and circle actions

Dedicated to Professor Shigeo Sasaki on his 60th birthday

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§ 0. Introduction.

Fix a compact Lie group G and a family $\mathcal{F}$ of subgroups of G. We consider all (φ, M) where M is a closed oriented smooth manifold, and $\varphi: G\times M\to M$ is an orientation preserving smooth G-action so that for $x\in M$ the isotropy subgroup

$$G_x=\{g\in G,\ \varphi(g, x)=x\}$$

is conjugate to a member of $\mathcal{F}$. Then a bordism group $\mathcal{O}_n(G;\mathcal{F})$ of $\mathcal{F}$-free oriented G-manifolds is defined.

Let S^1 be the unit circle in the field of complex numbers and regard it a compact Lie group. Let $Z_m=\{t\in S^1, t^m=1\}$ be the cyclic subgroup of order m.

Given an oriented Z_m-manifold (φ, M), consider a cartesian product $S^1\times M$ and let Z_m act on $S^1\times M$ by

$$t\cdot(z, x)=(zt^{-1}, \varphi(t, x))$$

for $t\in Z_m$, $z\in S^1$ and $x\in M$. Denote by $S^1\times_{Z_m}M$ the orbit manifold and by $[z, x]$ the point of $S^1\times_{Z_m}M$ represented by (z, x) of $S^1\times M$. Then there is a circle action $\bar{\varphi}$ on $S^1\times_{Z_m}M$ given by

$$\bar{\varphi}(\lambda, [z, x])=[\lambda z, x].$$

If (φ, M) is an oriented $\mathcal{F}$-free Z_m-manifold, then $(\bar{\varphi}, S^1\times_{Z_m}M)$ is $\mathcal{F}$-free S^1-manifold and this induces an extension homomorphism

$$\boldsymbol{E}:\ \mathcal{O}_n(Z_m;\mathcal{F})\longrightarrow\mathcal{O}_{n+1}(S^1;\mathcal{F}).$$

On the other hand, let $\mathcal{F}$ be a family of subgroups of S^1, denote by $\mathcal{F}_m$ the family of subgroups of Z_m given by

$$\mathcal{F}_m=\{Z_m\cap H,\ H\in\mathcal{F}\}.$$

Let (φ, M) be an oriented $\mathcal{F}$-free S^1-manifold, then the restriction $(\varphi|Z_m, M)$ is an oriented $\mathcal{F}_m$-free Z_m-manifold and this restriction induces a homomorphism