J. Math. Soc. Japan
Vol. 24, No. 2, 1972

On rank 3 groups with a multiply transitive constituent

By Eiichi BANNAI*)

(Received Sept. 1, 1971)

§ 1. Introduction.

We say that a permutation group $(\mathfrak{G}, \Omega)$ is a primitive extension of rank 3 of a permutation group (G, Δ) if the following conditions are satisfied: (i) $\mathfrak{G}$ is primitive and of rank 3 on the set Ω, and (ii) there exists an orbit $\Delta(a)$ of the stabilizer $\mathfrak{G}_a$ $(a \in \Omega)$ such that the action of $\mathfrak{G}_a$ on $\Delta(a)$ is faithful and that $(\mathfrak{G}_a, \Delta(a))$ and (G, Δ) are isomorphic as permutation groups.

The purpose of this note is to prove the following theorem:

THEOREM 1. *Let (G, Δ) be a 4-ply transitive permutation group. If (G, Δ) has a primitive extension of rank 3, then one of the following cases holds:*

(I) $|\Delta|=5$, $G=S_5$,

(II) $|\Delta|=7$, $G=S_7$ *or* A_7,

(III)[1] $|\Delta|=57$ *and* $G\neq S_{57}, A_{57}$,

where S_n and A_n denote the symmetric and alternating groups on Δ $(|\Delta|=n)$ respectively.

Theorem 1 is regarded as a sort of generalization of the results in T. Tsuzuku [6] and S. Iwasaki [3] where primitive extensions of rank 3 of symmetric and alternating groups are determined.

The author wishes to express his hearty thanks to Professor Ryuzaburo Noda for his kind criticisms, which have improved Theorem 1 to this general form.[2] The author also thanks Mr. Hikoe Enomoto for the valuable discussions we had.

§ 2. Proof of Theorem 1.

LEMMA 1. *Let $\mathfrak{G}$ be a primitive rank 3 permutation group on Ω, and let $\mathfrak{G}_a$ be doubly transitive on one of its orbits $\Delta(a)$. Let $\Gamma(a)$ be another orbit*

*) Supported in part by the Fujukai Foundation.

1) Professor Noboru Ito has kindly shown the author the proof of the non-existence of non-trivial 4-ply transitive permutation group of degree 57 in a letter dated on Aug. 18, 1971. Therefore the case (III) of Theorem 1 does not occur.

2) In the original manuscript Theorem 1 is proved with the additional hypothesis that the case (B) in the proof of Theorem 1 holds.