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Minimal submanifolds with *m*-index 2 and generalized Veronese surfaces

Dedicated to Professor Kentaro Yano on his 60th birthday

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For a submanifold M in a Riemannian manifold $\bar{M}$, the *minimal index* (*m-index*) at a point of M is by definition the dimension of the linear space of all the 2nd fundamental forms with vanishing trace. The *geodesic co-dimension* (*g-codim*) of M in $\bar{M}$ is defined by the minimum of codimensions of M in totally geodesic submanifolds of $\bar{M}$ containing M.

In [8] and [9], the author investigated minimal submanifolds with m-index 2 everywhere in Riemannian manifolds of constant curvature and gave some typical examples of such submanifolds with g-codim 3 and g-codim 4 in the space forms of Euclidean, elliptic and hyperbolic types. Each example is the locus of points on a moving totally geodesic submanifold intersecting orthogonally a surface at a point. This surface is called the base surface. This situation is quite analogous to the case of the right helicoid in E^3 generated by a moving straight line along a base helix.

When the ambient space is Euclidean, the base surface of the example in case of g-codim 4 is a minimal surface in a 6-sphere, whose equations are analogous to those of the so-called Veronese surface which is a minimal surface in a 4-sphere with m-index 2 and g-codim 2. In [2], T. Itoh gave a minimal surface of the same sort in an 8-sphere.

In the present paper, the author will give some examples of minimal submanifolds with m-index 2 and g-codim of any integer $\geqq 2$ in the space forms of Euclidean, elliptic and hyperbolic types. The base surfaces corresponding to the minimal submanifolds with m-index 2 and even geodesic codimension in Euclidean spaces will be called generalized Veronese surfaces.

§ 1. Preliminaries

Let $M=M^n$ be an n-dimensional submanifold of an $(n+\nu)$-dimensional Riemannian manifold $\bar{M}=\bar{M}^{n+\nu}$ of constant curvature $\bar{c}$. Let $\bar{\omega}_A$, $\bar{\omega}_{AB}=-\bar{\omega}_{BA}$, $A, B=1, 2, \cdots, n+\nu$, be the basic and connection forms of $\bar{M}$ on the orthonormal