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Local structures of groups of diffeomorphisms

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0° Introduction

Although it is well known that the groups $\mathcal{D}$ of the smooth diffeomorphisms of compact manifolds M are so called Frechet Lie groups [2, 3, 7], the category of Frechet Lie groups seems to the author to be still too huge to treat and get some useful results. Comparing with Hilbert Lie groups, Frechet Lie groups have not so nice property. This is mainly because the implicit function theorem or the Frobenius theorem does not hold in the category of Frechet manifolds in general case. As a matter of fact, these groups $\mathcal{D}$ have a little nicer property, which is an analogue with the Sobolev chain in function spaces. In fact, $\mathcal{D}$ is an inverse limit of a series of smooth Hilbert manifolds $\mathcal{D}^s$ [2, 7]. However, group operations are not so simple as additive groups of function spaces. These have a little more complicated property (see [2, 7]. Also this will be proved again in this paper as an immediate conclusion of Theorem B and Lemma 1). Anyway since here is a chain $\mathcal{D}^s$, it is natural to think that many properties of $\mathcal{D}$ (not $\mathcal{D}^s$ itself) can be proved by using this chain, and at this time, since we want to get properties of $\mathcal{D}$ (not $\mathcal{D}^s$), inequalities which are similar to Garding's inequality are becoming needful. Garding's inequality was necessary to prove the regularity of elliptic operators, and here to get properties of $\mathcal{D}$ we need some kinds of regularity theorems.

However, to get desired inequalities in general one has to write down all formulas explicitly without using local coordinates of M. This paper is one of the efforts of doing these. All results, especially several inequalities obtained in this paper, will be needful to prove the theorem which is mentioned in the introduction in [8]. To consider the properties of $\mathcal{D}$ many concepts and quantities defined on M must be lifted up to those of $\mathcal{D}$. At this point, the Theorems A, B which are mentioned below are becoming important. The purpose of this paper is to prove these theorems and as applications of these to prove some of local properties of $\mathcal{D}$ as well as some of inequalities. (See next section to know how these theorems are applied.)

To state the Theorems A, B we have to begin with the following: Let E and F be finite dimensional vector bundles over a compact n-dimensional