

On elliptic modular surfaces

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(Received March 1, 1971)

Introduction

The purpose of this paper is to study a certain class of algebraic elliptic surfaces called elliptic modular surfaces from both analytic and arithmetic point of view. Our results are based on the general theory of elliptic surfaces due to Kodaira [11].

Let B denote an (algebraic) elliptic surface having a global section over its base curve Δ . We denote by J and G the functional and homological invariants of B over Δ , and by $\mathcal{F}(J, G)$ the family of (not necessarily algebraic) elliptic surfaces over Δ with the functional and homological invariants J and G . We assume throughout the paper that J is non-constant and that the fibres of B over Δ contain no exceptional curves of the first kind. The part I is devoted to the generalities on such an elliptic surface B . In §1, we give an explicit description of the structure of the Néron-Severi group of B ; for the sake of later use, the results are formulated over an arbitrary algebraically closed ground field. In §2, we compute the cohomology groups $H^i(\Delta, G)$ of the base curve (or Riemann surface) Δ with coefficients in the sheaf G following Kodaira. This gives an analytic proof of the so-called Ogg-Šafarevič's formula. In §3, it is shown that, in the family $\mathcal{F}(J, G)$ of analytic elliptic surfaces, algebraic surfaces are "dense" (Theorem 3.2); this answers a question raised by Kodaira. The results in §1 or 2 must be well-known, but they are included here because we could not find a suitable reference.

In the part II, we develop the analytic theory of elliptic modular surfaces. First in §4 we define the elliptic modular surface B_Γ for each subgroup Γ of finite index of $SL(2, \mathbf{Z})$ such that $\Gamma \ni -1$, and examine its singular fibres and numerical characters. The base Δ_Γ of B_Γ is the compact Riemann surface associated with Γ . In §5, we show that the group of global sections of an elliptic modular surface B_Γ over Δ_Γ is a finite group (Theorem 5.1). In other words, the generic fibre of B_Γ is an elliptic curve defined over the field K_Γ of modular functions with respect to Γ , and it has only a finite

* Partially supported by Fujukai.