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An isometric immersion of a homogeneous Riemannian manifold of dimension 3 in the hyperbolic space

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Introduction.

In the previous paper [1], the present author investigated a homogeneous Riemannian manifold admitting an isometric immersion in the euclidean space or the hyperbolic space. But recently an error is pointed out by Prof. S. Tanno. Precisely the calculation at the first line in page 408 is incorrect. It must be corrected as follows;

$$0=d\omega_{12}=\Omega_{12}-\omega_{13}\wedge\omega_{32}=(K_1+K-b_3c_3)\omega_1\wedge\omega_2=(K_1+2K)\omega_1\wedge\omega_2 .$$

Hence if $K\neq 0$, there are no contradictions. So Lemma 3.5 and also Theorem B are valid only if the dimension of M is greater than 3.

The purpose of this paper is to determine a structure of a connected homogeneous Riemannian manifold of dimension 3 which admits an isometric immersion of the type number 2 in the 4 dimensional hyperbolic space.

Let G be a group of all matrices of the following type

$$\begin{pmatrix} e^t & 0 & -\xi \\ 0 & e^{-t} & \eta \\ 0 & 0 & 1 \end{pmatrix} \qquad \xi,\ \eta,\ t\in R .$$

Then (ξ, η, t) can be considered as a coordinate system of G. In this coordinate system we give a Riemannian metric on G by

$$ds^2=e^{-2t}d\xi^2+e^{2t}d\eta^2+dt^2 .$$

This metric is invariant by left translations of G and with this metric G can be considered as a homogeneous Riemannian manifold of dimension 3. In this paper we call this Riemannian manifold a B-manifold. (Although the definition of a B-manifold given in § 1 is different from the above definition, it will be shown in Theorem 1 that both definitions are equivalent.) The main results of this paper are the following:

(1) A connected homogeneous Riemannian manifold M of dimension 3 admits an isometric immersion in H^4 of the type number 2, if and only if