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Localization of CW-complexes and its applications

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Introduction

In the algebraic topology, in particular in the homotopy theory, abelian groups are often treated by being devided into their "p-primary component" for various primes p.

In the homotopy category of 1-connected CW-complexes, an isomorphism means a homotopy equivalence, which is of course an equivalence relation. As is well known, a homotopy equivalence is such a map that it induces an isomorphism on the integral homology group.

There might be three ways to generalize it in the mod p sense.

First one is to define a p-equivalence so that it induces an isomorphism on the homology group with Z_p-coefficient. A p-equivalence, however, is not in general an equivalence relation even in the category of 1-connected finite CW-complexes. In fact, in [**11**] is shown an example, for which symmetricity does not hold. To make it an equivalence relation, we have to work in the category of p-universal spaces [**12**].

Next one is to define that X and Y are of same p-type, if there exist a space Z and p-equivalences $f: X \to Z$ and $g: Y \to Z$. Then it is easy to see that a relation being of same p-type is an equivalence relation.

The last one is to consider a homotopy equivalence for "localized spaces $X_{(p)}$" of X at p. It is a functor of 1-connected CW-complexes into itself such that if $f: X \to Y$ is a p-equivalence then the localization at p $f_{(p)}: X_{(p)} \to Y_{(p)}$ is a homotopy equivalence. The localization is studied by Adams [**2**], Anderson [**3**], Bousfield-Kan and others. Our construction is a generalization of Adams' telescope [**2**], and has the following advantage:

THEOREM 2.5. *If X is a 1-connected CW-complex of finite type, then $H_*(X_{(p)}) \cong H_*(X) \otimes Q_p$ and $\pi_*(X_{(p)}) \cong \pi_*(X) \otimes Q_p$, where Q_p denotes the ring of those fractions, whose denominators, in the lowest form, are prime to p.*

Also we show

COROLLARY 4.3. *X is homotopy equivalent to $\prod_{X_{(0)}} X_{(p)}$ the pull back of $X_{(p)}$ over $X_{(0)}$.*

So we can study the topological properties of X for each prime p