J. Math. Soc. Japan
Vol. 23, No. 3, 1971

On the alternating groups IV

By Takeshi KONDO

(Received Oct. 20, 1970)

§ 0. Introduction.

The object of this note is to prove the following result.

THEOREM. *Let $\tilde{z}$ be an arbitrary involution of the alternating group $\mathfrak{A}_m$ of degree m. Let G be a finite group with the following properties:*

(i) *G has no subgroup of index 2,*

(ii) *G contains an involution z such that $C_G(z)$ is isomorphic to $C_{\mathfrak{A}_m}(\tilde{z})$.*

Then if $m \equiv 2$ or $3 \bmod 4$ and $m \geqq 7$, G is isomorphic to $\mathfrak{A}_m$.

Obviously this is a generalization of [**3**; Th. I] in which the author proved the theorem in the case where $\tilde{z}$ is an involution of $\mathfrak{A}_m$ with the longest cycle decomposition. In [**4**; Th. A], the author also proved that, in the case $m \equiv 0$ or $1 \bmod 4$, if $\tilde{z}$ is an involution of $\mathfrak{A}_m$ with the longest cycle decomposition, G is isomorphic to $\mathfrak{A}_m$ with a few exceptions in the case of small degrees. Of course, we can expect a generalization of this result similar to the theorem, but the author has not obtained any such results. The reason lies in the point that we cannot find out any method to examine the fusion of involutions.

The main work of this note is to examine the fusion of involutions of the groups which satisfy the conditions of the theorem. On the basis of these results, we can determine the precise structures of the normalizers of some elementary abelian subgroups. Then it turns out that this knowledge enables us to calculate the centralizers of involutions other than a given one in the exact same way as in [**3**; § 5 and § 6]. (So we shall omit the detailed discussions of this part.) Then, by applying our previous result [**3**; Th. I], we can obtain the theorem. Essentially the method to examine the fusion of involutions is also similar to our previous work [**5**], but, in some points, we need different kinds of arguments from [**5**]. So we shall discuss the examination of the fusion of involutions in full detail.

The notations and the terminologies which were introduced in the introduction of [**3**] or [**4**] will be freely used.