J. Math. Soc. Japan
Vol. 23, No. 3, 1971

Axiomatic theory of non-negative fullsuperharmonic functions

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(Received Aug. 11, 1970)

The axiomatic research of non-negative superharmonic functions on a harmonic space has been treated by M. Brelot, R. M. Herve, H. Bauer, and C. Constantinescu and A. Cornea. In the study of elliptic or parabolic differential equations we can find many applications of their theory ([**4**], [**10**]). Our motivation in the present paper is related to the study of elliptic differential equations with certain lateral conditions. What lateral conditions may be considered on a harmonic space? The problem of this sort was first considered by R. S. Martin in connection with the Dirichlet problem and by Z. Kuramochi in connection with the Neumann problem. In the case of axiomatic theory of harmonic functions K. Gowrisankaran made a study of the Dirichlet problem. The axiomatic formulation of Kuramochi's theory was given by F. Y. Maeda [**15**]. It seems that many known lateral conditions may be considered within the framework of *fullharmonic structure* introduced by Maeda. (We can give the fullharmonic structure that corresponds to the solutions of an elliptic differential equation with a Wentzel's boundary condition lacking the term that indicates, in a probability language, the jumps to the interior.)

Starting from a given fullharmonic structure on a Brelot's harmonic space, we shall make a research of many properties of fullsuperharmonic functions (that is to say, supersolutions of an elliptic differential equation with a lateral condition). Many properties of superharmonic functions that were studied extensively by Brelot and Herve are also valid for fullsuperharmonic functions (Minimum principle, etc.). They are studied by F. Y. Maeda. Maeda has proved a partition theorem, which, in the case of Brelot's theory, was obtained by Herve and is called by the name of Herve's partition theorem. We shall further develop his results, and shall construct potential kernels. We shall give a submarkov resolvent such that the excessive functions relative to this resolvent are exactly the non-negative fullsuperharmonic functions (Sections 3 and 4). P. A. Meyer is the first who constructed a resolvent such that the excessive functions relative to the resolvent are the non-negative superharmonic functions in the axiomatic theory of Brelot. This problem has been