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Existence of digital extensions of semi-modular state charts

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Let J and W be a finite set of indices and the set of non-negative integers, respectively. By W^J we mean the cartesian product of W, with itself $|J|$ times where $|J|$ is the number of elements of J. A semi-modular state chart (V, h) is said to be finite whenever the number of similarity classes of V is finite [**7**]. Moreover, we say (V, h) is digital if, for arbitrary M and N of V, $h(M)=h(N)$ implies $M \sim N$. A state chart (V^e, h^e) is called a digital extension of (V, h) when (V^e, h^e) is digital and restrictions $V^e|J$ and $h^e|J$ are equal to V and h, respectively. Then there exists a binary, distributive and digital extension of (V, h), if (V, h) is binary, finite and distributive [**8**]. The principal aim of the present paper is to generalize the above result under the condition of semi-modularity of (V, h), which is the affirmative solution of one of the fundamental problems proposed by D. E. Muller and W. S. Bartky [**1, 2, 3**] as a model of asynchronous circuits, and later mathematically reorganized by H. Noguchi [**5, 6, 7**] as a mathematical system constructed over relations. Terminology of the paper relies on [**5, 6, 7, 8**].

We prove the following theorem.

MAIN THEOREM. *A finite, binary and semi-modular state chart* (V, h) *is finitely realizable. In fact, there exists a distributive state chart* (D^e, h^e) *which induces a binary, semi-modular and digital extension* (V^e, h^e) *of* (V, h).

In § 1 a special type of extension called a separation is defined and a necessary and sufficient condition for existence of digital extensions is given (Theorem 1.2). Thus in order to prove Main Theorem we have only to show that there exists a separation of (V, h). However, in this paper we look for a wide class of digital extensions rather than giving a direct proof of Main Theorem. For this purpose, relations between semi-modular subsets and distributive subsets are investigated as follows; if V is semi-modular then $\sigma(V)$ becomes a change diagram, and $\mu(\sigma(V))$ is well defined and $(\mu(\sigma(V)), h)$ is finite if (V, h) is finite. It is to be noted that Corollary 1.13 suggests the idea of the proof of Main Theorem.

In § 2 some properties of the induced synthetic relation $\underset{\cdot}{\sim}$ and v-similarity