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Notes on minimal immersions

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§ 0. Introduction.

In this note we shall study minimal immersions of Riemannian manifolds in some Riemannian manifolds with certain properties. In particular, positions of compact minimal submanifolds (with oriented boundary or without boundary) in complete Riemannian manifolds with some curvature conditions will be our main concern.

One of the essential tools in our study here is an integral-geometric inequality, to the effect that if the Laplacian Δf of a smooth function f defined over a compact Riemannian manifold (without boundary) has definite sign, then f is constant everywhere, where the Laplace-Beltrami operator is taken with respect to the induced Riemannian metric on the minimal submanifold under consideration. Hermann [7] used this to prove a uniqueness theorem for minimal submanifolds in a complete Riemannian manifold of non-positive curvature, which is a clue for us to prove that a compact minimal submanifold in a product Riemannian manifold $V^k \times R^m$, V^k being compact, is contained in $V^k \times \{p\}$, $p \in R^m$.

Another tool is a well-known concavity property, used first by Tompkins [**19**], by which various results on isometric immersions are obtained, for example, when an ambient manifold is of non-positive curvature [**10**, **14**] or when an ambient manifold is of positive curvature and compact [**5**, **6**, **17**]. In this context, we shall study a compact minimal submanifold lying in some special position in a compact Riemannian manifold of positive curvature.

Throughout this note, we shall employ definitions and notation as those of [**3**]. In § 1, a proof of a theorem of Hermann will be given with some corrections to his original one. In § 2, we shall consider Hermann's condition in the case where the ambient manifold is of non-negative curvature and of product type $V \times R^m$. In § 3, the concavity condition for a compact minimal hypersurface in a Riemannian manifold of positive curvature will be considered.