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Correction to my paper: Conjugate classes of Cartan subalgebras in real semisimple Lie algebras

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In our previous paper [1], we stated in Theorem 7 (p. 415) that if $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ is a real semisimple Lie algebra of the first category, then for any maximal admissible (i.e. strongly orthogonal) subset $\boldsymbol{F}=\{\alpha_1, \cdots, \alpha_l\}$ of $\boldsymbol{R}_{\mathfrak{p}}$ (the set of non compact roots), the subspace

$$\mathfrak{m}(\boldsymbol{F})=\sqrt{-1}\sum_{i=1}^{l} R(E_{\alpha_i}+E_{-\alpha_i})$$

is a maximal abelian subalgebra in $\mathfrak{p}$. However this statement is false as the example at the end of this note shows. We shall prove in this note that Theorem 7 in [1] remains valid if we replace a maximal admissible subset of $\boldsymbol{R}_{\mathfrak{p}}$ by an admissible subset of $\boldsymbol{R}_{\mathfrak{p}}$ having the maximal number of elements. In the remaining part of [1] § 4, we used Theorem 7 to construct a maximal abelian subalgebra in $\mathfrak{p}$. As a matter of fact, all the maximal admissible sets in $\boldsymbol{R}_{\mathfrak{p}}$ used in [1] have the maximal number of elements. Therefore all the results in § 4 of [1] remain valid. We use the notation in § 4 of [1]. In particular let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the real semisimple Lie algebra and its Cartan decomposition, $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$ contained in $\mathfrak{k}$, $\boldsymbol{R}$ be the set of all roots with respect to $\mathfrak{h}^c$, E_α $(\alpha \in \boldsymbol{R})$ be a Weyl base corresponding to the compact form $\mathfrak{g}_u=\mathfrak{k}+i\mathfrak{p}$, that is, $\mathfrak{g}_u=\mathfrak{h}+\sum_{\alpha\in \boldsymbol{R}}\{R(E_\alpha+E_{-\alpha})+R\sqrt{-1}(E_\alpha-E_{-\alpha})\}$.

LEMMA 1. *The sum of three non compact positive roots is not a root.*

PROOF. Let $B=\{\alpha_1, \cdots, \alpha_r\}$ be the set of all simple roots with respect to the given linear order in $\boldsymbol{R}$ and $\beta=\sum_{i=1}^{r} m_i\alpha_i$ be the maximal root in $\boldsymbol{R}$. Any root $\alpha=\sum_{i=1}^{r} n_i\alpha_i$ satisfies the inequality

$$n_i \leqq m_i \quad (1 \leqq i \leqq r). \tag{1}$$

We can assume that $\mathfrak{g}$ is simple. In this case there exists only one non compact root, say α_j, in B. The coefficients of α_j in β satisfies

$$1 \leqq m_j \leqq 2. \tag{2}$$

The set $\boldsymbol{R}_{\mathfrak{p}}^+$ of all non compact positive roots is given by