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# On $D$-dimensions of algebraic varieties*)

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## § 1. Introduction.

Let $k$ be an algebraically closed field of characteristic zero. We shall work in the category of schemes over $k$. Let $V$ be a complete algebraic variety, and let $D$ be a divisor on $V$. In this paper, we shall introduce the notion of the $D$-dimension of $V$ which we denote by $\kappa(D, V)$, and prove some theorems (Theorems 1, 2, 3 and 4) about $\kappa(D, V)$. Furthermore, when $V$ is non-singular, we define the *Kodaira dimension* (or the canonical dimension) $\kappa(V)$ of $V$, to be $\kappa(K_V, V)$, where $K_V$ denotes a canonical divisor of $V$. The Kodaira dimension would seem to be the most fundamental invariant in the theory of birational classification of algebraic varieties. Our theorems concerning $\kappa(D, V)$ and $\kappa(V)$ establish fundamental results in the theory of birational classification. In particular, Theorem 5 shows that it would be enough to consider algebraic varieties of Kodaira co-dimension zero[1], of Kodaira dimension zero and of Kodaira dimension $-\infty$, in order to classify algebraic varieties to the extent that Italian algebraic geometers did for algebraic surfaces about sixty years ago.

The main results of this paper have been announced in [9].

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## § 2. Statement of the results.

Letting $V$ be a complete algebraic variety of dimension $n$ and $D$ a divisor on $V$, we denote by $l(D)-1$ the dimension of the complete linear system $|D|$ associated with $D$. We consider the set of all positive integers $m$ satisfying $l(mD) > 0$, which we indicate by $N(D)$. Assume that $N(D)$ is not empty. Then $N(D)$ forms a sub-semigroup of the additive group of all integers. Hence,

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1) The Kodaira co-dimension of an algebraic variety $V$ of dimension $n$ is defined to be $n-\kappa(V)$.