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On the index of a semi-free S^1-action

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§1. Introduction.

Let G be a compact Lie group, M^n a closed smooth n-manifold and $\varphi: G\times M^n\rightarrow M^n$ a smooth action. Then the fixed point set is a disjoint union of smooth k-manifolds F^k, $0\leqq k\leqq n$.

P.E. Conner and E.E. Floyd [2] obtained several properties of fixed point sets of smooth involutions and one of their results is the following.

Suppose that $T: M^{2k}\rightarrow M^{2k}$ is a smooth involution on a closed manifold of odd Euler characteristic. Then some component of the fixed point set is of dimension $\geqq k$.

Now we consider semi-free smooth S^1-actions on oriented manifolds and the purpose of this paper is to show the following results.

THEOREM 1.1. *Let M^n be an oriented closed smooth n-manifold and $\varphi: S^1\times M^n\rightarrow M^n$ a semi-free smooth action. Then each k-dimensional fixed point set F^k can be canonically oriented and the index of M^n is the sum of indices of F^k, that is,*

$$\boldsymbol{I}(M^n)=\sum_{k=0}^{n}\boldsymbol{I}(F^k)\,.$$

THEOREM 1.2. *Suppose that $\varphi: S^1\times M^{4k}\rightarrow M^{4k}$ is a semi-free smooth S^1-action on an oriented closed manifold of non-zero index. Then some component of the fixed point set is of dimension $\geqq 2k$.*

§2. Semi-free S^1-action.

Let S^1 and D^2 denote the unit circle and the unit disk in the field of complex numbers. Regard S^1 as a compact Lie group. Let M^n be an oriented closed smooth n-manifold and $\varphi: S^1\times M^n\rightarrow M^n$ a smooth action. The action φ is called semi-free if it is free outside the fixed point set. Then we have the following ([4], Lemma 2.2).

LEMMA 2.1. *The normal bundle of each component of the fixed point set in M^n has naturally a complex structure, such that the induced S^1-action on this bundle is a scalar multiplication.*

From this lemma, a codimension of each component of the fixed point set