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Satake compactification and the great Picard theorem

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§ 1. Introduction.

Let $\varDelta$ be the unit disk $\{z \in \boldsymbol{C};\ |z|<1\}$ in the complex plane and $\varDelta^*$ the punctured disk $\{z \in \boldsymbol{C};\ 0<|z|<1\}$. Let $P_1(\boldsymbol{C})$ be the 1-dimensional complex projective space, $P_1(\boldsymbol{C}) = \boldsymbol{C} \cup \{\infty\}$. Delete three points, say, $0, 1, \infty$, from $P_1(\boldsymbol{C})$. The great Picard theorem says that every holomorphic mapping $f: \varDelta^* \to P_1(\boldsymbol{C}) - \{0, 1, \infty\}$ can be extended to a holomorphic mapping $f: \varDelta \to P_1(\boldsymbol{C})$.

We consider a generalization of the great Picard theorem. Given a complex space M, let d_M be the intrinsic pseudo-distance introduced in [3]. We say that M is hyperbolic if d_M is a distance on M. For example, $P_1(\boldsymbol{C}) - \{0, 1, \infty\}$ is hyperbolic. Consider the following question.

" Let Y be a complex space and M a complex hyperbolic subspace of Y such that its closure $\bar{M}$ is compact. Does every holomorphic mapping $f: \varDelta^* \to M$ extend to a holomorphic mapping $f: \varDelta \to Y$? "

The answer is, in general, negative as shown by Kiernan [2] (see also [4, Ch. VI, § 1]). On the other hand, we have the following result, [4].

THEOREM 1. *Let Y be a complex space and M a complex subspace of Y satisfying the following conditions:*

(1) *M is hyperbolic;*

(2) *the closure $\bar{M}$ of M is compact;*

(3) *Given a point p on the boundary $\partial M = \bar{M} - M$ and a neighborhood $\mathcal{U}$ of p, there exists a smaller neighborhood $\mathcal{V}$ of p in Y such that*

$$d_M(M \cap (Y - \mathcal{U}), M \cap \mathcal{V}) > 0 .$$

Let X be a complex manifold and A a locally closed complex submanifold of X. Then every holomorphic mapping $X - A \to M$ extends to a holomorphic mapping $X \to Y$.

It has been shown in [4; Ch. VI, § 6] that if $Y = P_2(\boldsymbol{C})$ and $M = P_2(\boldsymbol{C}) - Q$, where Q is a complete quadrilateral, then the three conditions of Theorem 1 are satisfied. Hence, every holomorphic mapping of $X - A$ into $P_2(\boldsymbol{C}) - Q$ extends to a holomorphic mapping of X into $P_2(\boldsymbol{C})$. This may be considered

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