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# On radii of convexity and starlikeness of some classes of functions

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## §1. Introduction.

Let $S_{\beta}^{*}$ be the class of functions

$$F(z)=z+a_2z^2+\cdots, \tag{1.1}$$

which are regular, univalent and starlike of order $\beta$ $(0\leqq\beta<1)$ in the unit circle $E$ $(|z|<1)$. Let $H_{\beta}^{*}$ be the class of functions

$$f(z)=\frac{1}{z}+b_0+b_1z+\cdots, \tag{1.2}$$

which are regular, univalent and starlike of order $\beta$ $(0\leqq\beta<1)$ in $0<|z|<1$. Let $V_{\beta}$ be the class of functions

$$g(z)=\frac{1}{2}[F(z)+zF'(z)], \tag{1.3}$$

where $F(z)\in S_{\beta}^{*}$. Let $T$ be the class of functions

$$F(z)=z+a_2z^2+\cdots,$$

which are regular, univalent and starlike in $|z|<1$ and satisfy the condition

$$\left|\frac{zF'(z)}{F(z)}-\alpha\right|<\alpha, \qquad \left(\alpha>\frac{1}{2}\right) \qquad \text{for } |z|<1. \tag{1.4}$$

If $P_{\beta}$ is the class of regular, analytic functions in $E$ $(|z|<1)$ whose real part is not less than $\beta$ and which take the value 1 at the origin, then it is easily seen that for every $u(z)\in P_{\beta}$ there exists a unique $p(z)\in P_0$ such that

$$u(z)=\beta+(1-\beta)p(z). \tag{1.5}$$

Hence, according as $h(z)\in S_{\beta}^{*}$ or $H_{\beta}^{*}$ we have

$$1+\frac{zh''(z)}{h'(z)}=\pm[\beta+(1-\beta)p(z)]+\frac{(1-\beta)zp'(z)}{\beta+(1-\beta)p(z)}, \tag{1.6}$$

negative sign pertaining to the case where $h(z)\in H_{\beta}^{*}$. Thus, in order to find the radius of convexity for the class $S_{\beta}^{*}$ or $H_{\beta}^{*}$ one needs to find the extreme values of