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Some closed subalgebras of measure algebras and a generalization of P. J. Cohen's theorem

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§ 0. Introduction.

Let G_1 and G_2 be locally compact abelian groups, and let $L^1(G_1)$ and $M(G_2)$ be the group algebra of G_1 and the measure algebra of G_2, respectively. Homomorphisms of $L^1(G_1)$ into $M(G_2)$ have been studied by H. Helson, W. Rudin, J. P. Kahane, Z. L. Leibenson, P. J. Cohen and others; and P. J. Cohen [1], [2] determined all the homomorphisms of $L^1(G_1)$ into $M(G_2)$ by the notion of the coset ring and piecewise affine maps. He also proved that every homomorphism of $L^1(G_1)$ into $M(G_2)$ has a natural norm-preserving extension to a homomorphism of $M(G_1)$ into $M(G_2)$, but in general an extension to a homomorphism of $M(G_1)$ into $M(G_2)$ is not unique.

The purpose of this paper is to introduce some closed subalgebra $L^*(G_1)$ of $M(G_1)$, which contains $L^1(G_1)$ properly if G_1 is not discrete, to determine the maximal ideal space of $L^*(G_1)$, and to determine all the homomorphisms of $L^*(G_1)$ into $M(G_2)$ as a generalization of P. J. Cohen's theorem.

We give in § 1 some preliminaries, and in § 2 we introduce a closed subalgebra $L^*(G_1)$ of $M(G_1)$. In § 3 we investigate the maximal ideal space of $L^*(G_1)$, and obtain it as a semi-group. Finally we determine in § 4 all the homomorphisms of $L^*(G_1)$ into $M(G_2)$ as a generalization of P. J. Cohen's theorem.

§ 1. Preliminaries.

Throughout this paper G_1 and G_2 denote locally compact abelian groups ($=LCA$ groups), and Γ_1 and Γ_2 denote their dual groups, respectively. The notations G^τ and Γ_τ are also used to express an LCA group with underlying group G and topology τ, and its dual group, respectively. Thus by G^τ and $G^{\tau'}$, we mean that they have the same underlying group G.

$L^1(G_1)$ is the group algebra of G_1, i. e. the Banach algebra of all the Haar integrable functions on G_1 under convolution multiplication, and $M(G_2)$ is the measure algebra of G_2, the Banach algebra of all the regular bounded complex