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Pseudogroups associated with the one dimensional foliation group (I)

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INTRODUCTION

Recent authors dealing with the classification of pseudogroups have treated the case where the group of first order terms is irreducible. Here the simplest of the reducible cases is treated; a classification is given of all pseudogroups whose first order terms generate the entire one dimensional foliation group $\begin{pmatrix} a & b_i \\ 0 & d^i{}_j \end{pmatrix}$.

These pseudogroups are classified modulo equivalence by coordinate changes in R^N; there are then 19 major classes of such pseudogroups, of which 2 are families of pseudogroups parameterized by a real constant, and 3 are families parameterized by a positive integral constant.

The classification proceeds in two steps. Step I is a classification of certain graded Lie algebras; such algebras appear in most pseudogroup theories. Step II is a solution of the equivalence problem and the actual derivation of the pseudogroups; here use is made of a theory due to R.C. Gunning at Princeton of pseudogroups defined by differential equations with constant coefficients.

Since Gunning has not published a detailed description of his theory, the first chapter of this paper contains a summary of his results. This theory is more restrictive than many recent treatments, but has the advantage of keeping formulas attractively explicit. Gunning considers the terms of order $\leqq n$ in the power series expansions of C^∞ functions at some fixed point x; these form the elements of a group $\tilde{G}^n_x$. Those terms in $\tilde{G}^n_x$ coming from the expansion of the functions of a pseudogroup Γ form a subgroup G^n_x of $\tilde{G}^n_x$. If the differential equations defining Γ have constant coefficients, this subgroup G^n does not vary over the manifold, and so can be used to define Γ. Hence Γ can be analyzed indirectly using Lie group methods on G^n.

For simplicity, Gunning omits terms of order 0; thus all pseudogroups are transitive. If local coordinates are chosen, the group $\tilde{G}^n$ can be given very explicitly. In particular, $\tilde{G}^1$ is just $GL(N, R)$, and G^1 is precisely the