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Local existence and analyticity of hyperfunction solutions of partial differential equations of first order in two independent variables

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§ 1. Introduction.

Let P be a differential operator of first order in two independent variables x and y,

$$P=a(x, y)\frac{\partial}{\partial x}+b(x, y)\frac{\partial}{\partial y}+c(x, y).$$

Here we assume that the coefficients a, b and c are (complex-valued) real analytic functions defined in an open set Ω in $\boldsymbol{R}^2$, and that

$$|a(x, y)|+|b(x, y)|\neq 0.$$

In this paper we shall study conditions for the local existence and analyticity of hyperfunction solutions of the equation $Pu=f$. The basic facts about the theory of hyperfunctions may be found in [2], [4]. We denote by $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{O}$ the sheaves of real analytic functions, hyperfunctions, and holomorphic functions, respectively.

Let p be the principal part of P. We define the k-th commutator c_p^k of p by induction:

$$c_p^0=\bar{p}=\text{the operator with complex conjugate coefficients},$$

$$c_p^k=[p, c_p^{k-1}]=pc_p^{k-1}-c_p^{k-1}p.$$

Let $k_p(x, y)$ denote the first value of k for which c_p^k is not proportional to p at the point (x, y). If c_p^k is proportional to p for all values of k, we define $k_p(x, y)$ to be ∞. Note that P is elliptic at (x, y), if and only if $k_p(x, y)=0$. It is easily seen that $k_p(x, y)$ does not depend on the choice of local coordinates, and that it is invariant under multiplication of P by a non-vanishing function.

Our main results are the following two theorems which state the relation between the parity of $k_p(x, y)$ and the analyticity and existence of hyperfunction solutions of $Pu=f$.