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On the isometry groups of Sasakian manifolds

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§1. Introduction

The dimension of the isometry group of an m-dimensional Riemannian manifold (M, g) is equal to or smaller than $m(m+1)/2$. The maximum is attained if and only if (M, g) is of constant curvature and one of the following manifolds: a sphere S^m, a real projective space P^m, a Euclidean space E^m, and a hyperbolic space H^m (cf. S. Kobayashi and K. Nomizu [**5**], p. 308).

G. Fubini's theorem ([**2**], or [**1**]; p. 229) says that in an m-dimensional Riemannian manifold $(m>2)$ the dimension of the isometry group can not be equal to $m(m+1)/2-1$. Further, by H. C. Wang [**12**] and K. Yano [**13**] it was shown that in an m-dimensional Riemannian manifold $(m \neq 4)$, there exists no group of isometries of order s such that

$$m(m+1)/2 > s > m(m-1)/2+1 . \tag{1.1}$$

Riemannian manifolds admitting isometry groups of dimension $m(m-1)/2+1$ were studied by K. Yano [**13**], and the related subjects were studied by S. Ishihara [**4**], M. Obata [**7**], etc.

We consider similar problems in Sasakian manifolds. For a Sasakian manifold M with structure tensors (ϕ, ξ, η, g) we denote by $I(M)$ and $A(M)$ the group of isometries and the group of automorphisms. By $S^{2n+1}[H]$ for $H>-3$, $E^{2n+1}[-3]$, and $(L, CD^n)[H]$ for $H<-3$, we denote complete and simply connected Sasakian manifolds of $(2n+1)$-dimension with constant ϕ-holomorphic sectional curvature $H>-3$, -3, and $H<-3$, respectively (S. Tanno [**11**]). These Sasakian manifolds admit the automorphism groups of the maximum dimension $(n+1)^2$ (cf. S. Tanno [**10**]). By $F(t)$ we denote the cyclic group generated by $\exp t\xi$ for a real number t. Manifolds are assumed to be connected and structure tensors are assumed to be of class C^∞.

In this paper the main theorem is as follows:

THEOREM A. *Let (M, ϕ, ξ, η, g) be a complete Sasakian manifold of m-dimension, $m=2n+1$.*

(i) *If $\dim I(M)=(n+1)^2$, then (M, ϕ, ξ, η, g) is one of the following manifolds:*