

The central limit theorem for additive functionals of Markov processes and the weak convergence to Wiener measure

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The first aim of this paper is to discuss the central limit theorem for additive functionals of conservative strong Feller processes on compact spaces. Secondly, as a refinement of the limit theorem, we shall consider a convergence theorem of measures on $C[0, T]$ formed by certain continuous additive functionals. Then the limit is the Wiener measure, that is, we shall deal with the so-called "invariance principle".

The central limit theorem of this type has been investigated by many authors. Fruitful results were obtained by S. V. Nagaev [7], I. S. Volkov [11], J. Keilson and D. M. G. Wishart [5], and others for discrete time Markov processes. In the case of continuous time Markov processes with finite state spaces, M. Fukushima and M. Hitsuda [3] gave the central limit theorem and some applications. Moreover, our central limit theorem is related to other types of limit theorems. In particular, it seems that the limit theorems for a stationary process under quite general conditions (Yu. A. Davydov, I. A. Ibragimov, M. I. Gordin, V. N. Soley [1]) are very close to our theorems, where some of our additive functionals can be considered as stationary processes.

The content of this paper :

In §1, we shall give a basic lemma related to the Fourier transform of the semigroup, and state some results on the spectral theory of operators. In §2, the central limit theorem (Theorem 2.1) will be established and we shall give the class of the exceptional additive functionals for which the "asymptotic variance" degenerates (Theorem 2.2). §3 will be devoted to the proof of the invariance principle (Theorem 3.2). Finally, we shall investigate the central limit theorem for additive vectors in §4, where the results are analogous to the case of one-dimensional additive functionals.

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