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On the generation of semigroups of nonlinear contractions

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Introduction

Let X be a real or complex Banach space and S be a subset of X. Let $\{T(t); t \geqq 0\}$ be a one-parameter family of (possibly nonlinear) contractions from S into itself satisfying the following conditions:

(i) $T(0)=I$ (the identity mapping), $T(t)T(s)=T(t+s)$ on S for $t, s \geqq 0$;

(ii) for each $x \in S$, $T(t)x$ is strongly continuous in $t \geqq 0$. Then the family $\{T(t)\}$ is called a *semigroup* (*of contractions*) *on* S. And we define the *infinitesimal generator* A_0 of a semigroup $\{T(t)\}$ by $A_0 x = \lim_{h \to +0} h^{-1}\{T(h)x-x\}$ and the *weak infinitesimal generator* A' by $A'x = w\text{-}\lim_{h \to +0} h^{-1}\{T(h)x-x\}$, if the right sides exist, the notation "lim" (or "w-lim") means the strong limit (or the weak limit) in X.

The purpose of the present paper is to construct the semigroup of contractions determined by a (nonlinear) operator given in a Banach space. Our results consist of sufficient conditions for a (multi-valued) operator in X or a pseudo-resolvent of contractions in X to determine a semigroup of contractions. Also, we are concerned with the generation of semigroups of differentiable operators.

We find other interesting results on the generation of semigroups of contractions in [2]-[5], [8]-[14], in which (multi-valued) maximal dissipative, m-accretive or m-dissipative operators are treated as the infinitesimal generators. In this paper we extend these generation theorems to the case of a (multi-valued) dissipative operator A such that the range $R(I-\lambda A)$ of $I-\lambda A$ contains $D(A)$ for every $\lambda > 0$. Recently, Brezis and Pazy [1] considered similar problems in Hilbert spaces. A result related to their generation theorem will be given in § 6.

Section 0 gives the notion of a dissipative operator and some of its basic properties.

Section 1 contains the statements of main results and some remarks.

Section 2 concerns the abstract Cauchy problem.