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On complex hypersurfaces of the complex projective space II

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§ 1. Introduction

Let $P_{n+1}(C)$ be the complex projective space of complex dimension $n+1$ with the Fubini-Study metric of constant holomorphic sectional curvature 1 and let M be a complex hypersurface of $P_{n+1}(C)$ with the induced Kaehler structure. The purpose of this paper is to prove the following theorem.

THEOREM. *Let M be a complete complex hypersurface of $P_{n+1}(C)$. If $n \geqq 2$ and if every sectional curvature of M is greater than* 1/8, *then M is a complex hyperplane $P_n(C)$.*

Postponing the proof of the theorem to the following section, we shall list here results in the same direction. For the sake of simplicity, we shall adopt the following notations: for example,

$K > \delta$: every sectional curvature of M is greater than δ,

$H > \delta$: every holomorphic sectional curvature of M is greater than δ.

A. *If M is complete and if $K \geqq \frac{1}{4}$, then $M = P_n(C)$ provided $n \geqq 2$.*

In a recent paper ([**5**]), K. Nomizu proved (A) in case of $n \geqq 3$. But (A) is an immediate consequence of the following well known results ([**1**], [**2**], [**7**]):

(a) $H \leqq 1$ for a complex hypersurface of $P_{n+1}(C)$.

(b) If $H \geqq 0$, then a maximum curvature is holomorphic.

(c) If $n \geqq 2$ and if $\delta \leqq K \leqq 1$, then $\frac{\delta(8\delta+1)}{1-\delta} \leqq H$.

The assumption of (A), together with (a) and (b), implies $\frac{1}{4} \leqq K \leqq 1$ and hence (a) and (c) imply $H = 1$ so that $M = P_n(C)$.

In [**6**] we proved

B. *If M is complete and if $H > \frac{1}{2}$, then $M = P_n(C)$.*

Let $z_0, z_1, \cdots, z_{n+1}$ be a homogeneous coordinate system of $P_{n+1}(C)$ and let $Q_n(C) = \{(z_0, \cdots, z_{n+1}) \in P_{n+1}(C) \mid \sum z_i^2 = 0\}$. Then it is known that $\frac{1}{2} \leqq H \leqq 1$

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