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A theorem in the theory of definition

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The well-known theorem of Beth on definability can be extended in different directions. One was pursued by Svenonius [**1**], another by Kueker [**2**]. By using the extended form of preservation theorems developed in Motohashi [**5**], Weglorz [**4**], we shall get an extension of Beth's theorem of a new kind in this paper.

§ 0. Preliminaries

We shall use the ordinary set-theoretical and model-theoretical notations (see [**3**], [**5**]). In this paper, we shall be concerned with the first order predicate calculus with equality $\doteqdot$, (abbr. by f.p.c.), $L, L', \cdots$, will be used to denote f.p.c. For L, $M(L)$ is the class of all the first order structures related to L.

Let $M = \bigcup_L M(L)$. For $L, L', L \subset L', L \cap L'$ have obvious meanings. L_0 denotes the f.p.c. without logical constants. Therefore $M(L_0)$ is the class of all non empty sets. Let $L \subset L'$, $\mathfrak{A} \in M(L')$, then $\mathfrak{A} \upharpoonright L$ means the reduct of $\mathfrak{A}$ to L, $|\mathfrak{A}|$ is the universe of $\mathfrak{A}$, and $\bar{\bar{\mathfrak{A}}} = |\bar{\bar{\mathfrak{A}}}|$. If $\gamma_1, \gamma_2, \cdots, \gamma_m$ are non logical constants, then $L(\gamma_1, \cdots, \gamma_m)$ is the f.p.c. having $\gamma_1, \cdots, \gamma_m$ as non logical constants in addition to those of L. For $\mathfrak{A}, \mathfrak{B} \in M(L)$ and $f \in |\mathfrak{B}|^{|\mathfrak{A}|}$, f is said to be an *embedding* of $\mathfrak{A}$ to $\mathfrak{B}$ if f is an injection and the image of $\mathfrak{A}$ by f is the substructure of $\mathfrak{B}$. For $\mathfrak{A} \in M(L)$, $L(\mathfrak{A})$ means the diagram language of $\mathfrak{A}$.

We assume that the reader is familiar with the notion of special models (see Morley-Vaught [**3**]).

§ 1. Main theorem

An operation $\mathcal{M}$ defined on M^2 is said to be a *morphism on models* (*m.o.m.*) if $\mathcal{M}(\mathfrak{A}, \mathfrak{B}) \subset |\mathfrak{B}|^{|\mathfrak{A}|}$ for $\mathfrak{A}, \mathfrak{B} \in M$.

For *m.o.m.* $\mathcal{M}$ and L, we define $\Delta_L(\mathcal{M})$ by the set of all formulas $F(v_0, v_1, \cdots, v_n) \in \mathfrak{F}(L)$ such that $\mathfrak{A} \models F[a_0, a_1, \cdots, a_n]$ implies $\mathfrak{B} \models F[f(a_0), f(a_1), \cdots, f(a_n)]$, for any $\mathfrak{A}, \mathfrak{B} \in M(L)$, $f \in \mathcal{M}(\mathfrak{A}, \mathfrak{B})$, $\langle a_0, a_1, \cdots, a_n \rangle \in |\mathfrak{A}|^{n+1}$.

DEFINITION. Let $\mathcal{M}$ be a *m.o.m.*