J. Math. Soc. Japan
Vol. 22, No. 3, 1970

Homogeneous hypersurfaces in spaces of constant curvature

By Tsunero TAKAHASHI

(Received Oct. 6, 1969)

Introduction.

S. Kobayashi proved [4] that a connected compact homogeneous Riemannian manifold of dimension n is isometric to the sphere if it is isometrically imbedded in the Euclidean space E^{n+1} of dimension $n+1$. T. Nagano and the present author proved [5] that a connected homogeneous Riemannian manifold M of dimension n is isometric to the Riemannian product of a sphere and a Euclidean space if M is isometrically imbedded in the Euclidean space E^{n+1} and the rank of the second fundamental form (which is called the type number in this paper) is not equal to 2 at some point.

One of the purposes of the present paper is to consider the case which was not treated in [5], that is, the case of the type number being equal to 2.

In this paper we consider the isometric immersion of a connected homogeneous Riemannian manifold of dimension n not only in a Euclidean space E^{n+1}, but also in a hyperbolic space H^{n+1} and we determine all the types of M.

Let $S^m(K)$ denote an m-dimensional sphere of radius $1/K$ for a positive constant K and $H^m(K)$ denote an m-dimensional hyperbolic space of negative curvature K for a negative constant K.

The underlying manifold of $H^m(K)$ is that of a Euclidean space E^m and the Riemannian metric of $H^m(K)$ is given by

$$ds^2=\sum_i (dx_i)^2+\frac{K}{1-K\sum_i (x_i)^2}(\sum_i x_i dx_i)^2 .$$

The main theorems are the following:

THEOREM A. *If a connected homogeneous Riemannian manifold M of dimension n admits an isometric immersion f in a Enclidean space E^{n+1}, M is isometric to $S^m\times E^{n-m}$ $(0\leqq m\leqq n)$. If the type number of f is greater than* 1 *at a point, f is an imbedding.*

THEOREM B. *If a connected homogeneous Riemannian manifold M of dimension n admits an isometric immersion f in a hyperbolic space $H^{n+1}(K)$ of curvature $K(<0)$, the type number $t(p)$ of f is either constantly equal to n or*