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Remarks on the validity of Hasse's norm theorem

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Introduction.

Let k be an algebraic number field. It is well-known that the obstruction to the validity of Hasse's norm theorem for a finite Galois extension K/k is described as a factor group of certain cohomology group ([2], Th. 20.6). T. Ono has noticed that there is a very close connection between the validity of Hasse's norm theorem and the Tamagawa number of the torus $R^{(1)}_{K/k}(G_m)$ ([6], n°6).

In this paper, we extend slightly the problem to the case of an arbitrary finite extension L/k. Thus the problem becomes the following one; If an element x of k is "local norm" at every place $\mathfrak{p}$, that is, if $x \in NL^*_{\mathfrak{p}}$ for every $\mathfrak{p}$, where $NL^*_{\mathfrak{p}}$ is the subgroup of $k^*_{\mathfrak{p}}$ generated by $N_i L^*_{\mathfrak{q}_i}$, then is x contained in NL^*? Note that $\mathfrak{q}_i$ runs all places of L above $\mathfrak{p}$, and that N_i is the norm map of $L_{\mathfrak{q}_i}$ into $k_{\mathfrak{p}}$. This problem is affirmatively solved for any cubic extension of k (n°3, Example). But we do not know for which type of extension of k this problem can be solved affirmatively.

In our paper, we denote by V the torus $R^{(1)}_{K/k}(G_m)$, and by U the torus whose character module is the dual of that of V. These tori can be defined in general situation. It is comparatively easy to calculate the Tamagawa number of U (n°3, Prop. 4). Following Ono's method, we calculate the Tamagawa number of V (n°4, Th.).

It is probable that our results can be expressed in terms of cohomology groups. But it seems to the author that our method in this paper is useful in the theory of non-Galois extensions of fields of dimension one.

1. Preliminaries.

Let G be a finite group and H be its subgroup of index n. One puts $G = \bigcup_{i=0}^{n-1} g_i H$ with $g_0 = 1$ (the identity of G). We consider the following left G-module:

(1) $$\Lambda = \boldsymbol{Z}[G/H],$$