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Cut elimination theorem for second order arithmetic with the Π_1^1-comprehension axiom and the ω-rule[1]

By Mariko YASUGI

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Introduction.

In [1] Schütte introduced the constructive ω-rule to first order arithmetic and proved the (complete) cut elimination theorem for the first order arithmetic, by translating it into a cut-free subsystem of the system with the constructive ω-rule. Takeuti extended this idea in [6] and showed that second order arithmetic with the Π_1^1-comprehension axiom can be translated into a cut free subsystem of second order arithmetic with the Π_1^1-comprehension axiom and the constructive ω-rule. This was done by modifying his consistency proof of the system **SINN** (cf. [5]), using the same system of ordinal diagrams.

In this article we shall prove the (complete) cut elimination theorem for second order arithmetic with the Π_1^1-comprehension axiom and the (general) ω-rule, using all countable ordinals. The proof of the theorem indicates that the reduction method which is used for the consistency proof of **SINN** works for the system with an infinite rule as well, although the system of ordinal diagrams which corresponds to the latter is no longer constructive.

At the end, we remark that if we restrict the ω-rule to the constructive one, then the cut elimination theorem holds within the system with the constructive ω-rule.[2]

§ 1. The formulation of the system.

In this section the system of second order arithmetic with the Π_1^1-comprehension axiom and the ω-rule is formulated. It is a modification of the system **SINN** in [5] and shall be called the system $\mathfrak{W}$.

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