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On the relatively cyclic imbedding problem with given local behavior

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Introduction

We shall assume that the reader is familiar with the paper [**1**].

Let Ω be an algebraic number field, and k a finite Galois extension of Ω with Galois group $\mathfrak{g}$. As in [**1**], let $(k/\Omega, G, \varphi)$ be the imbedding problem associated with an exact sequence of finite groups

$$1 \longrightarrow A \longrightarrow G \overset{\varphi}{\longrightarrow} \mathfrak{g} \longrightarrow 1\,. \tag{1}$$

For each prime $\mathfrak{p}$ of Ω, we choose a prime $\mathfrak{P}$ in k lying above $\mathfrak{p}$ and fix it once and for all. Usually we shall denote the $\mathfrak{P}$-adic completion $k_{\mathfrak{P}}$ by $k^{\mathfrak{p}}$. Let $\mathfrak{g}^{\mathfrak{p}}$ be the local Galois group $G(k^{\mathfrak{p}}/\Omega_{\mathfrak{p}})$ and put $G^{\mathfrak{p}}=\varphi^{-1}(\mathfrak{g}^{\mathfrak{p}})$. Then we have an exact sequence

$$1 \longrightarrow A \longrightarrow G^{\mathfrak{p}} \overset{\varphi^{\mathfrak{p}}}{\longrightarrow} \mathfrak{g}^{\mathfrak{p}} \longrightarrow 1\,.$$

Here, $\varphi^{\mathfrak{p}}$ denotes the restriction of φ to $G^{\mathfrak{p}}$.

Let E be a finite set of primes of Ω, and suppose that we are given a solution $K(\mathfrak{p})$ of $(k^{\mathfrak{p}}/\Omega_{\mathfrak{p}}, G^{\mathfrak{p}}, \varphi^{\mathfrak{p}})$ for each prime $\mathfrak{p}\in E$. We say that the imbedding problem with given local behavior

$$(k/\Omega, G, \varphi\,; K(\mathfrak{p}), \mathfrak{p}\in E)$$

is solvable, if there exists a solution K of $(k/\Omega, G, \varphi)$ with the following properties:

1) The algebra K is a field.
2) The algebra $K_{\mathfrak{P}}$ $(=k^{\mathfrak{p}}\otimes_k K)$ is identified with $K(\mathfrak{p})$ as Galois algebras for each $\mathfrak{p}\in E$.

In this paper we shall treat this problem in case A is a cyclic group. Since it will be shown that this problem can be reduced to the case where A has a prime power order l^n, and further to the case where we can suppose that k contains a primitive l^n-th root of unity ζ, we can restrict our attention to that case.