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On some doubly transitive permutation groups of degree n and order $2^l(n-1)n$

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1. Introduction.

Doubly transitive permutation groups of degree n and order $2(n-1)n$ were determined by N. Ito ([**9**]). Some doubly transitive permutation groups of degree n and order $4(n-1)n$ were studied in [**10**].

The object of this paper is to prove the following result.

THEOREM. *Let Ω be the set of symbols* $1, 2, \cdots, n$. *Let $\mathfrak{G}$ be a doubly transitive group on Ω of order $2^l(n-1)n$ $(l>1)$ not containing a regular normal subgroup and let $\mathfrak{K}$ be the stabilizer of symbols* 1 *and* 2. *Assume that $\mathfrak{K}$ is cyclic. Then $\mathfrak{G}$ is isomorphic to one of the groups $PGL(2, *)$, $PSL(2, *)$, $PSU(3, 3^2)$ and $PSU(3, 5^2)$.*

We use the standard notation. $C_{\mathfrak{X}}(\mathfrak{T})$ denotes the centralizer of a subset $\mathfrak{T}$ in a group $\mathfrak{X}$ and $N_{\mathfrak{X}}(\mathfrak{T})$ stands for the normalizer of $\mathfrak{T}$ in $\mathfrak{X}$. $\langle S, T, \cdots\rangle$ denotes the subgroup of $\mathfrak{X}$ generated by elements $S, T, \cdots$ of $\mathfrak{X}$.

2. On the degree of the permutation group $\mathfrak{G}$.

1. Let $\mathfrak{H}$ be the stabilizer of the symbol 1. $\mathfrak{K}$ is of order 2^l and it is generated by a permutation K. Let us denote the unique involution $K^{2^{l-1}}$ of $\mathfrak{K}$ by τ. Since $\mathfrak{G}$ is doubly transitive on Ω it contains an involution I with the cyclic structure $(1\ 2)\cdots$. Then we have the following decomposition of $\mathfrak{G}$;

$$\mathfrak{G}=\mathfrak{H}+\mathfrak{H}I\mathfrak{H}.$$

Since I is contained in $N_{\mathfrak{G}}(\mathfrak{K})$, it induces an automorphism of $\mathfrak{K}$ and (i) $K^I=K$ or $K\tau$, (ii) $K^I=K^{-1}\tau$ or (iii) $K^I=K^{-1}$. (For the case $l=2$, (i) $K^I=K$ or (iii) $K^I=K^{-1}$.) If an element $H'IH$ of a coset $\mathfrak{H}IH$ of $\mathfrak{H}$ is an involution, then $IHH'I=(HH')^{-1}$ is contained in $\mathfrak{K}$. Hence, in the case (i) the coset $\mathfrak{H}IH$ contains just two involutions, namely $H^{-1}IH$ and $H^{-1}\tau IH$, in the case (ii) it contains just 2^{l-1} involutions, namely $H^{-1}K'IH$ for $K'\in\langle K^2\rangle$, and in the case

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