J. Math. Soc. Japan
Vol. 22, No. 2, 1970

On the theory of commutative formal groups

By Taira HONDA

(Received Nov. 10, 1969)

The theory of (commutative) formal groups was initiated by M. Lazard and J. Dieudonné around 1954. Lazard [**11**], [**12**] studied commutative formal groups over an arbitrary commutative ring by treating the coefficients of power series explicitly. Whereas Dieudonné investigated formal groups over a field of characteristic $p>0$ exclusively. He reduced in [**4**] the study of commutative formal groups over a perfect field of characteristic $p>0$ to that of modules over a certain non-commutative ring, so-called Dieudonné modules, and obtained in [**5**] a complete classification of isogeny classes of commutative formal groups over an algebraically closed field of characteristic $p>0$. Later Manin [**16**] studied isomorphism classes of simple formal groups. The study of one-dimensional formal groups over $\mathfrak{p}$-adic integer rings was begun by Lubin [**13**] and a number of interesting results were obtained by him and Tate.

In this paper we first construct a certain general family of commutative formal groups of arbitrary dimension over a $\mathfrak{p}$-adic integer ring. Over the ring $W(k)$ of Witt vectors over a perfect field of characteristic $p>0$, this exhausts all the commutative formal groups. These are attached to a certain type of matrices with elements in the ring $W(k)_\sigma[[T]]$ of non-commutative power series, where σ is the Frobenius of $W(k)$, and homomorphisms of these formal groups are described in terms of matrices over $W(k)_\sigma[[T]]$. By reducing the coefficients of formal groups over $W(k)$ mod $pW(k)$ we get formal groups over k. It is shown that all the commutative formal groups over k are obtained in this manner. Moreover homomorphisms of commutative formal groups over k are also described in terms of $W(k)_\sigma[[T]]$-modules by lifting these homomorphisms to power series over $W(k)$. Thus we get the main results of Dieudonné [**4**] again by the method quite different from his. In [**4**] he used tools peculiar to characteristic $p>0$ and his construction of formal groups was indirect, whereas in our method the relation between formal groups over $W(k)$ and those over k is transparent and the construction of formal groups is explicit and elementary.

We now explain briefly how to construct commutative formal groups over $W(k)$ in case of dimension one. Take an element u of $W(k)_\sigma[[T]]$ of the