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On infinitesimal automorphisms of Siegel domains

Dedicated to Prof. Atuo Komatu for his 60th birthday

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Introduction

Let D be the Siegel domain of the second kind in the space $\boldsymbol{C}^N$ of N $(=n+m)$ complex variables due to Pyatetski-Shapiro [6], associated with a convex cone V in the space $\boldsymbol{R}^n$ of n real variables and a V-hermitian form F on the space $\boldsymbol{C}^m$ of m complex variables. By an infinitesimal automorphism of the domain D, we mean a holomorphic vector field X on D which is complete, that is, generates a global one parameter group φ_t of transformations.

The main purpose of the present paper is to give the details of the results announced in the note [8], establishing some theorems on the Lie algebra $\mathfrak{g}$ of all infinitesimal automorphisms of a Siegel domain D of the second kind.

Assume that the domain D is affine homogeneous. At the outset we prove that the Lie algebra $\mathfrak{g}$ is endowed with the structure of a graded Lie algebra as follows: $\mathfrak{g}=\sum_{p=-\infty}^{\infty}\mathfrak{g}^p$ (direct sum); $[\mathfrak{g}^p, \mathfrak{g}^q]\subset\mathfrak{g}^{p+q}$; $\mathfrak{g}^p=\{0\}$ $(p<-2)$ and the subalgebra $\mathfrak{g}_a=\mathfrak{g}^{-2}+\mathfrak{g}^{-1}+\mathfrak{g}^0$ of $\mathfrak{g}$ is just the Lie algebra of all infinitesimal affine automorphisms of D (Theorem 3.1). Then we prove that the graded Lie algebra $\mathfrak{g}_a$ is prolonged to a graded Lie algebra $\hat{\mathfrak{g}}=\mathfrak{g}^{-2}+\mathfrak{g}^{-1}+\mathfrak{g}^0+\sum_{p=1}^{\infty}\hat{\mathfrak{g}}^p$ and that the graded Lie algebra $\mathfrak{g}$ is determined as a suitable graded subalgebra of $\hat{\mathfrak{g}}$ (Theorem 4.1). From a geometric point of view, the Lie algebra $\hat{\mathfrak{g}}$ may be described as a Lie algebra of polynomial vector fields X on $\boldsymbol{C}^N$ tangent to the Silov boundary S of the domain D (See § 4). In [6], Pyatetski-Shapiro has determined the graded Lie algebra $\mathfrak{g}_a$ in terms of the cone V and the V-hermitian form F. Theorem 4.1 in turn enables us to compute the Lie algebra $\mathfrak{g}$ on the basis of the Lie algebra $\mathfrak{g}_a$ (See § 5, Examples).

In our discussion, it is important that every infinitesimal automorphism X on the domain D is extended to a holomorphic vector field defined on the whole $\boldsymbol{C}^N$ and tangent to the real submanifold S of $\boldsymbol{C}^N$ (Proposition 3.1). Owing to this fact, our problems can be connected, to a great extent, with the geometry of real submanifolds of complex manifolds and hence with the geometry of differential systems as developed by Tanaka [9].