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Orbits of one-parameter groups I

(Plays in a Lie algebra)

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§ 1. Introduction.

By an analytic group and by an analytic subgroup of a Lie group, we mean a connected Lie group and a connected Lie subgroup of a Lie group, respectively. Unless specified otherwise, an analytic subgroup and its corresponding Lie subalgebra will be denoted by the same capital script and capital Roman letter, respectively. For example, if $\mathcal{G}$ denotes an analytic group and $\mathcal{L}$ denotes an analytic subgroup of $\mathcal{G}$, then G will denote the Lie algebra of $\mathcal{G}$, and L will denote the subalgebra of G corresponding to $\mathcal{L}$. We make the convention that the Lie algebra of a Lie group is the tangent space of that group at the identity.

The fields of real numbers and complex numbers will be denoted by $\boldsymbol{R}$ and $\boldsymbol{C}$, respectively.

Throughout this paper, we shall adopt the terminology and utilize the theorems in

N. Jacobson, *Lie algebras*, Tracts in Math. 10, Interscience, 1962,

and

C. Chevalley, *Theory of Lie groups* I, Princeton, 1946.

In particular, § 2 and § 3 below are connected with the former, and § 4 and § 5 with the latter.

Let $\mathcal{G}$ be an analytic group, and let $\mathcal{L}$ be an analytic subgroup of $\mathcal{G}$. Let X and Y be elements of G. The purpose of this paper is to give a necessary and sufficient condition, in terms of Lie algebras, for the validity of the equality $\exp \boldsymbol{R}X \cdot \mathcal{L} = \exp \boldsymbol{R}Y \cdot \mathcal{L}$. In particular, if $\mathcal{L}$ is a closed subgroup, this equality implies that the orbits of one-parameter groups $\exp \boldsymbol{R}X$ and $\exp \boldsymbol{R}Y$, passing through the point $\mathcal{L}$, coincide in the factor space $\mathcal{G}/\mathcal{L}$.

In order to explain our results, we first adopt the notation $(ad\ A)B = [A, B]$ for elements A and B of a Lie algebra, and introduce the following

DEFINITION. Let G be a Lie algebra, and let L be a subalgebra of G.

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