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Double ruled surfaces and their canonical systems*)

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Generally we shall follow the definitions and notations in Weil [4] and we shall consider projective varieties exclusively. Thus *varieties* are projective varieties, and surfaces and curves are (projective) varieties of dimension two and one respectively. To state our results, we first recall and introduce several definitions. k denotes, once and for always, an algebraically closed subfield of the field of complex numbers.

DEFINITION. (i) A variety U is a *rational variety over* k if and only if U is birationally equivalent over k to a projective space $\boldsymbol{P}_n$. (ii) A surface S is a *ruled surface over* k *with the base* B if and only if S is birationally equivalent over k to the product of the projective line $\boldsymbol{P}_1$ and a curve B defined over k. (iii) A surface S is a *double ruled surface over* k *with the base* B if and only if there is a rational mapping defined over k of degree two of S to a ruled surface $\boldsymbol{P}_1 \times B$ over k. S is a *double plane over* k if and only if there is a rational mapping defined over k of degree two of S to a rational surface over k (or the projective plane). (iv) We say that $\pi: S \to B$ is a *pencil over* k *of curves* or S has a pencil over k of curves if and only if S is a non-singular surface defined over k, B a non-singular curve defined over k, π a morphism defined over k, and a generic fibre $F_b = pr_S[\Gamma_\pi \cdot (S \times b)]$, $b \in B$, is irreducible (a curve defined over $k(b)$).

The purpose of this note is to find the image S_K of a double ruled surface S over k under the rational mapping induced by the canonical system. It turns out that, if S_K is of dimension two, then it is a ruled surface *over* k (Theorem 1); in particular we see that, if S is a double plane over k, then the image S_K is a *rational variety over* k (Corollary 2 to Theorem 1). These results remind us of a well-understood property of the canonical system of hyperelliptic curves. On the way to reach Theorem 1, the following results are proven and used. Proposition 2 generalizes, in some sense, Lüroth's Theorem to the effect that if a surface is the image of a rational mapping

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