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Noninvariant hypersurfaces of almost contact manifolds

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§ 1. Introduction.

A hypersurface of an almost contact manifold does not in general possess an almost complex structure as is seen by the example of S^4 in R^5 or in S^5. When considered as a unit sphere in R^6, S^5 carries a contact metric structure with respect to which S^4 cannot be imbedded as an invariant hypersurface. In fact, Theorem 5 says that it is impossible to imbed a manifold as an invariant hypersurface of a contact space. This situation is in marked contrast with the well-known fact that a hypersurface (real codimension 1) of an almost complex manifold admits an almost contact structure. However, this hypersurface is clearly not invariant, since the real codimension is 1, for, otherwise it admits an almost complex structure.

We are thereby led to consider noninvariant hypersurfaces of almost contact manifolds M. These again admit almost complex structures, but, in addition, there is a distinguished 1-form α induced by the contact form of M. This situation is examined in detail when the ambient space is affinely cosymplectic.

The metric case is especially interesting. Indeed, if M is quasi-Sasakian (e.g., a normal contact or cosymplectic space) and P is a noninvariant hypersurface, then P carries a symplectic, in fact, a Kaehlerian structure, with Kaehler metric γ.

§ 2. Hypersurfaces of almost contact manifolds.

Let $M(\phi, \xi, \eta)$ be a $(2n+1)$-dimensional almost contact manifold whose structure is defined by a linear transformation field ϕ acting in each tangent space M_m of M, $m \in M$, a vector field ξ on M and a *contact form* η such that

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