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Embedding and Existence theorems of infinite Lie algebra

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In [3] and [5], V. W. Guillemin, I. M. Singer and S. Sternberg gave Existence and Uniqueness theorem and Realization theorem concerning the abstract transitive Lie algebra. In this paper we shall give some extensions of these theorems, i. e. Embedding theorem (Section 5) and Existence theorem (Section 7). The former involves as its applications Realization theorem and also theorems concerning the graded Lie algebra, the latter may be said to be a relative existence theorem with the higher order structure constant. We begin by giving an abstract definition of an infinite Lie algebra and a truncated Lie algebra of any order. Roughly speaking, we shall construct Lie algebra as a projective limit of truncated Lie algebras. By this method, we can simplify the proofs, especially of Existence theorem, and also state the properties of the higher order structure constant. (It is shown in Section 3 that our infinite Lie algebra is equivalent to the complete filtered Lie algebra [4], and hence also to the abstract transitive Lie algebra.)

1. Throughout this paper, all vector spaces and Lie algebras are assumed to be defined over a commutative field of characteristic 0. Suppose that a collection of

a sequence of finite dimensional vector spaces $V_0, V_1, \cdots, V_n, \cdots$,

a sequence of maps $0 \xleftarrow{\pi_0} V_0 \xleftarrow{\pi_1} V_1 \longleftarrow \cdots \xleftarrow{\pi_n} V_n \longleftarrow \cdots$, and

a sequence of bracket products $[\ ,\ ]'_n \colon V_n \times V_n \ni (x, y) \mapsto [x, y]'_n \in V_{n-1}$, $n = 0, 1, 2, \cdots, (V_{-1} = 0)$ is given, and that the following conditions (a)—(f) are satisfied for all $n \geqq 1$.

(a) π_n is linear and surjective;

(b) $\pi_{n-1}[x, y]'_n = [\pi_n x, \pi_n y]'_{n-1}$, for all $x, y \in V_n$;

(c) $[\ ,\ ]'_n$ is bilinear and anti-symmetric;

(d) $J'_n(x, y, z) = 0$ for all $x, y, z \in V_n$, where J'_n is a trilinear anti-symmetric map of $V_n \times V_n \times V_n$ into V_{n-2} defined by

$$J'_n(x, y, z) = [[x, y]'_n, \pi_n z]'_{n-1} + [[y, z]'_n, \pi_n x]'_{n-1} + [[z, x]'_n, \pi_n y]'_{n-1};$$